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Final Report

WORKSHOP ON ADAPTIVE CONTROL

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May 8-10, 1979
University Inn, Champaign, Illinois

J. B. Cruz, Jr.
Workshop Organizer and Chairman

Sponsored by

The Air Force Office of Scientific Research
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DYNAMIC SYSTEMS
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FOREWORD

The Workshop on Adaptive Control was held to assess the state of the art in adaptive control research. It was felt that with the recent advances in control theory research along with the revolution in microprocessor capabilities that real adaptive control systems may be a possibility in the near future. A second objective (although not in sequential order) of the workshop was to identify future research topics. The right mixture of practitioners and theoreticians provided the right catalyst to answer both objectives. This copy of the final report represents the results of that workshop.

I wish to express special thanks to Dr. J.B. Cruz, Jr. of Dynamic Systems for organizing the workshop. However, his excellent job would not have been evident without the involvement of the participants. To the participants I say thanks for a job well done.

CHARLES L. NEFZGER, Major, USAF

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A workshop on Adaptive Control, with thirty four participants, examined the status of the field, discussed potential applications, and agreed on directions for future research. Three working groups were formed, one on robust control, one on model reference adaptive control and self-tuning regulators, and one on stochastic adaptive control. Areas were identified where concepts from robust control would combine with those from active adaptive control to provide a powerful approach. Numerous suggestions for future research were generated.

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ABSTRACT

A workshop on Adaptive Control, with thirty-four participants, examined the status of the field, discussed potential applications, and agreed on directions for future research. Three working groups were formed, one on robust control, one on model-reference adaptive control and self-tuning regulators, and one on stochastic adaptive control. The three working groups met separately as well as jointly. Areas were identified where concepts from robust control would combine with those from active adaptive control to provide a powerful approach. Numerous suggestions for future research were generated.

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I. INTRODUCTION

A. Background

Recent advances in adaptive control theory, stochastic control theory, computational algorithms for control, and computer technology point towards the feasibility of controllers for navigation, guidance, and flight control systems which function over a wide range of adverse operating conditions. Furthermore, recent results indicate that fixed structures with no adaptation are capable of operating over a wide range of conditions. One purpose of the workshop was to provide a forum for assessing the state of the art and for relating recent theory to current control problems in aerospace technology and other fields. A second purpose of the workshop was to have an open discussion of future directions in research on adaptive control, stimulated in part by projected needs.

The field of control exists because of the need to operate a system satisfactorily in spite of uncertainty associated with the dynamic system to be controlled. The uncertainty comes about because of inaccuracies in modeling, changes in environmental conditions, and presence of unavoidable disturbance inputs. The cornerstone of control theory is feedback theory, which originated with electronic amplifiers [1-3]. The first books on control systems [4,5] and several dozen others written in the last 30 years rely on the potential benefits of feedback to counteract the effects of uncertainty. With increased complexity, multiplicity of feedback loops, and large parameter variations, the exploitation of feedback is not easy but still possible [6]. In addition to stabilization, reduction of sensitivity to parameters, reduction of nonlinear distortion, and reduction of effects of disturbance inputs, feedback may be useful for maintaining near-optimality for a range of parameters [7].

When the changes in parameters are large or when there are extreme variations in environmental conditions such as in aircraft control over a wide range of flight conditions, a fixed control may not be adequate. Starting in the mid fifties self-adaptive controls whose control parameters change in consonance with changes in the controlled process have been proposed [8,9]. In the last twenty years there have been significant advances in adaptive control.

Feedback systems with fixed controllers were called passive adaptive systems [8] in the early days of adaptive control. Many advances have occurred in passive adaptive systems and there is a resurgence of interest in this subject. The present-day terminology for this approach is robust control. Since this is a simpler control structure, it is an attractive alternative to active adaptive control whenever the control problem could be solved using a robust control.

In much of robust control and adaptive control, the uncertainty is described in terms of deterministic concepts. Alternatively, these parameter uncertainties could be modeled probabilistically. Furthermore, the disturbance inputs could be modeled as stochastic processes. Such an approach leads to the theory of stochastic control [10,11]. A special class of stochastic control problems has been successfully solved using the method of self-tuning regulators [12]. The more general problem is much more complex and thus far, the available results are at a conceptual and theoretical level [13]. There is much current work on obtaining simpler algorithms and perhaps in a few years, there would be some substantial applications.

B. Organization of Workshop

The workshop concentrated on three topics: a) robust control, b) model reference adaptive control and self-tuning regulators, and c) stochastic adaptive control. Although self-tuning regulators are special stochastic adaptive controls, they were lumped with model reference adaptive controls because of recent results unifying these two areas. There were thirty-four participants from industry, universities, and government laboratories. The first session was devoted to a presentation of three key papers, one from each of the discussion topics. The next session was devoted to a general discussion of the three topics. The second day was devoted to separate discussions of the three topics. The workshop participants were divided into three working groups with a discussion leader for each. The last session on the third day was devoted to a presentation of the conclusions of the three groups.

All participants were urged to prepare short statements on their starting points for the discussions. The submitted individual contributions, as well as the three key papers, are included in this report. The working groups were asked to examine the state of the art of the subfields, to examine application areas where the present results might be used, and to explore future research directions. The three working groups were:

<u>Working Group on Robust Control</u>

J. Ackermann, Discussion Leader	R. K. Mehra
D. Bowser	C. L. Nefzger
D. P. Looze	W. R. Perkins
R. G. Marsh	M. G. Safonov
J. Medanic	K. K. D. Young

Working Group on Model Reference Adaptive Control and Self-Tuning Regulators

I. D. Landau, Discussion Leader	G. Kreisselmeier
G. F. Franklin	L. Ljung
C. A. Harvey	R. V. Monopoli
C. R. Johnson	A. S. Morse
H. Kaufman	K. S. Narenda
P. V. Kokotović	E. G. Rynaski

Working Group on Stochastic Adaptive Control

Y. Bar-Shalom, Discussion Leader	C. S. Padilla
P. E. Caines	T. Riggs
J. B. Cruz, Jr.	A. V. Sebald
J. Dillow	E. C. Tacker
B. Friedland	L. Tesfatsion
D. G. Lainiotis	P. Vergez

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II. KEYNOTE PAPERS

ROBUST CONTROL

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on leave from DFVLR Oberpfaffenhofen

1. Robust Control Problems

Robustness of control systems is defined in terms of system properties, which are invariant under a specified class of perturbations. Typical examples of desirable system properties are:

1) Stability or nice stability (e.g. defined by constraints on eigenvalue locations).
2) Limited deterioration of a performance index.
3) Limited deviation from an ideal behavior, e.g. constraints on step responses or frequency responses or on the return difference.
4) Limited deviation from a reference behavior, e.g. deviation from a nominal trajectory or a reference model response.
5) Tracking, i.e. zero asymptotic error for a class of reference and disturbance inputs.
6) Limited demand on control $|u|$ and control rate $|\dot{u}|$.

Perturbations may occur in the structure or in the parameters of a system. Examples of structural perburbations are:

1) Sensor failures.
2) Actuator failures.
3) Switching from automatic to manual control. Here it is desirable that the operator sees a stable system whenever he opens one or more feedback loops.
4) Change in system order due to a failure. Example: An aggregate

description for several power generators or a traffic flow or economic variables must be dissolved into a more detailed description of transients between individual components in failure situations.

Parametric perturbations are due to uncertainties in the plant model and in the controller implementation. Examples are:

5) Analytically known dependence of a plant model on uncertain physical parameters. Example: The linearized equations of a crane with physical parameters m_c = crab mass, m_l = load mass, ℓ = rope length, g = gravitational constant, and state variables x_1 = crab position, x_2 = crab velocity, x_3 = rope angle and x_4 = rope angular velocity are

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & m_l g/m_c & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & 0 \end{bmatrix} \underline{x} + \frac{1}{m_c} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1/\ell \end{bmatrix} u \qquad (1)$$

with $\omega^2 = (m_c + m_l)g/m_c \ell$. Input u is the force accelerating the crab. The crane may operate with an unknown load mass m_l between the empty hook and the maximum mass, for which the crane is designed. It may also operate with an unknown constant rope length between zero and the height of the crane.

6) Numerically known dependence of a plant model on uncertain physical parameters. Example: linearized equations of longitudinal motion of an aircraft depending on altitude and speed

$$\dot{\underline{x}} = \underline{A}_j \underline{x} + \underline{B}_j \underline{u} \qquad (2)$$

with J typical flight conditions, $\underline{A}_j$, $\underline{B}_j$, j = 1,2,...,J, in the flight envelope.

7) Known dynamics, which have disappeared in a simplified design model by linearization, truncation of structural modes, model reduction, neglecting of actuator and sensor dynamics. In some cases it may be possible to pull out all uncertainties as illustrated by Fig. 1, where for P = 0 the nominal plant N is obtained.

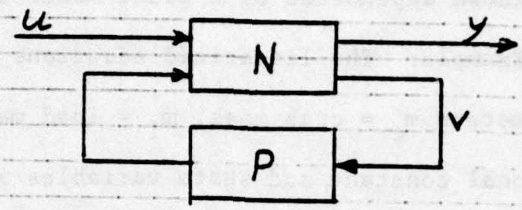

Fig. 1. Nominal plant N with perturbations P.

If the perturbations P can be expressed as a diagonal matrix of linear or nonlinear operators, then a robust design must provide sufficient gain and phase margins for the individual loops opened at v.

8) Unknown dynamics, which cannot be modeled. In this case only vague assumptions about perturbations δA, δB, δC of the system matrices A, B, C ($\dot{x} = Ax + Bu$, $y = Cx$) or perturbations $\delta G(s)$ of the transfer function matrix G(s) can be made.

9) Quantization effects and time delays in controller implementation.

10) Variance of components in mass produced control systems and circuits.

These lists of system properties and perturbations show, that many special combinations can be specified. Therefore many different definitions of "robust control" can be found in the literature.

The design problem for a robust control system may be formulated in one of the following three forms:

1. Given a system property. Which is the class of perturbations with respect to which the system property is robust? Design the controller such that the class of admissible perturbations is extended in the direction of the really expected perturbations.
2. Given a class of perturbations. Which maximum deviation from a desired system behavior occurs under the worst perturbation in the given class. Design the controller such that the maximum deviation is minimized.
3. Given a system property and a class of perturbations. Does there exist a set of controllers for which the system property is robust under the class of perturbations? If yes, select one on other criteria than robustness. If no, relax specifications.

2. Controller Structure

Design problems for control systems are usually parameterized by the assumption of a controller structure which defines a vector of design parameters. With the availability of cheap computers the main constraints from the side of controller implementation are given by the available actuators and sensors. However another constraint is given by the required time for a particular design. This of course depends on the available design methods and software for it.

Two typical assumptions for the controller structure are adaptive controllers or fixed gain controllers (e.g. state feedback, dynamic output feedback). One extreme is the attempt to obtain as much information about the perturbations as possible by on-line identification and failure detection. Then ideally the structure and parameters of the controller are adapted in order to achieve the best possible performance of the control system given the

momentarily available information. An intrinsic difficulty of this approach is that plant inputs, which admit a fast and accurate identification, are not good to achieve the best performance and vice versa. Also a tradeoff between a <u>fast</u> failure detection, identification and adaptation and a <u>reliable</u> one, which avoids false alarms and noise sensitivity of the adaptation, must be made.

The other extreme is the attempt to find a fixed gain controller which accomodates a specified class of perturbations. In this approach it may be necessary to sacrifice some performance in the nominal case in order to achieve robustness for the perturbed situation. Only this case is usually called robust control, however it should be apparent from the previous discussion, that robustness is a desirable feature also for an adaptive control system. The fixed gain solution indicates whether a more complex adaptive system is needed at all, or how far one has to go adaptive. Practical solutions to the robustness problem will frequently be in between the two extremes. They may also employ variable structures with state dependent switching between fixed linear feedbacks [30,31].

Frequency domain design techniques admit any dynamical order of the controller. State space design methods usually assume a state feedback controller structure. For a system with known parameters, all information which is relevant for its future dynamic is contained in the present state vector. If the full state is available, then the processing of past states, e.g. in dynamic feedback elements, cannot improve the performance of the system. If, however, the system parameters are perturbed, information about their actual values is contained in past states; adaptive systems make use of this fact. In this situation also the performance of fixed gain controllers can

be improved by processing past states in a dynamic controller, e.g. in dynamic output feedback. Uncertain parameters may also be introduced as additional states, which may be estimated and fed back.

In the following various robustness problems will be discussed, for which at least partial solutions are available in the literature.

3. Sensitivity, Robustness With Respect to Small Parametric Perturbations

3.1. Frequency Domain Methods

The main reasons for the use of feedback are stabilization and the preservation of desirable system properties in spite of noise inputs and perturbations of system parameters.

The reduction of nonlinear distortions was an essential reason for the use of feedback amplifiers, see Black [1]. The reduction of nonlinearity by high gain feedback has been further investigated by Cruz [2] and Desoer and Wang [3].

In frequency design methods the concept to compensate the loop, such that high gains are possible without instability, is the classic rule of thumb for the reduction of noise and uncertainty. Bode [4] expressed it in terms of gain and phase margins and a sensitivity function, which was generalized to the multivariable case by Cruz and Perkins [5]. A sensitivity matrix S(s) relates the output errors $E_c(s)$ due to perturbations in a feedback system to the output errors $E_o(s)$ due to the same perturbations in a corresponding open loop system by $E_c(s) = S(s)E_o(s)$. The sensitivity matrix S(s) is the inverse of the return difference matrix, for the loop of Fig. 2.

$$S(s) = [I + G(s)K(s)H(s)]^{-1} \tag{3}$$

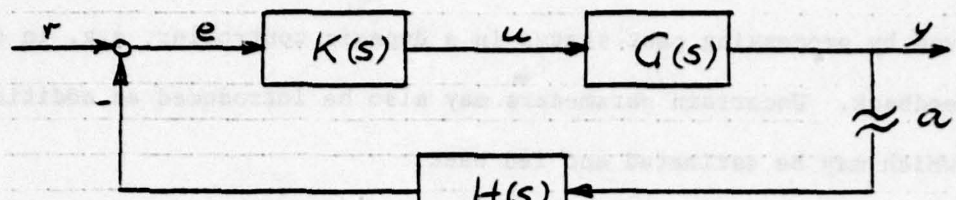

Fig. 2. Feedback system, return difference for loops broken at a.

Note that G(s) is the actual plant, which may be expressed by the nominal design model $G_N(s)$ and a perturbation $\delta G(s)$, i.e. $G(s) = G_N(s) + \delta G(s)$. If the known $G_N(s)$ is used in eq. (3) instead of the unknown G(s), then all results are local, i.e. restricted to small $\delta G(s)$. For a reduction of sensitivity it is sufficient that

$$S^T(-j\omega)S(j\omega) - I \leq 0 \quad \text{(neg. semidefinite)} \tag{4}$$

over the frequency band of interest, or in terms of the return difference $F(s) = I + G(s)K(s)H(s)$

$$F^T(-j\omega)F(j\omega) - I \geq 0 \,. \tag{5}$$

Hsu and Chen [6] proved the relationship

$$\det F(s) = \frac{\text{closed loop characteristic polynomial}}{\text{open loop characteristic polynomial}} \,. \tag{6}$$

Thus, if no cancellations occur, closed loop stability can be analyzed using det F(s). MacFarlane [7] studied the eigenvalues $\rho_j(s)$, $j = 1,2,\ldots,m$ of F(s) and showed that the closed loop is stable, if all characteristic frequency loci $\rho_j(j\omega)$, $j = 1,2,\ldots,m$ satisfy the Nyquist criterion. He also proved a necessary condition for the system to be optimal in the sense of a quadratic criterion $\int_0^\infty (y^TQy + u^TRu)dt$, it is

$$|\rho_j(j\omega)| \geq 1 \quad \text{for } 0 \leq \omega \leq \infty \quad j = 1,2,\ldots,m \tag{7}$$

or

$$|\det F(j\omega)| \geq 1 \quad \text{for all } \omega . \tag{8}$$

These results have the graphical interpretation that the complex plane plots of $|\det F(j\omega)|$ or $|\rho_j(j\omega)|$ must not penetrate the interior of the unit disc. It follows from this that the characteristic frequency loci of an optimal proportional feedback controller have infinite gain margin and at least 60^o phase margin.

Robustness of stability with respect to gain and phase changes may also be achieved in design by Rosenbrock's inverse Nyquist array [8]. Here $I + G_o^{-1}(j\omega)$ with $G_o(s) = G(s)K(s)H(s)$, see Fig. 2, is analyzed graphically and modified in the design. A standard technique in multivariable control system design is to use compensation or feedback to decouple or approximately decouple a multivariable system into several single input systems, which may be designed by single-loop techniques. Rosenbrock [8] uses the criterion of diagonal dominance for approximate decoupling.

Doyle showed by counterexamples [9] that these methods can lead to highly optimistic margins for individual loop gains, even if only very small margins exist for simultaneous change of several loop gains. Already in the single-input case, gain and phase margins are insufficient to characterize what happens for simultaneous gain and phase perturbations. Another difficulty is that by compensation or feedback for diagonal dominance the actual location of the uncertainty is obscured.

Doyle [9] examines the properties of the return difference using the concepts of singular values, singular vectors and the spectral norm of a matrix. The singular values σ_i of a matrix A are the non-negative square roots of the eigenvalues of A*A, where A* is the conjugate transpose of A. Since A*A is Hermitian, its eigenvalues are real. The singular values give a measure of

how close A is to being singular. The ratio of the smallest singular value $\underline{\sigma}$ and the largest one, $\overline{\sigma}$, is the condition number $\underline{\sigma}/\overline{\sigma}$. One may also interpret the singular values as generalizing to matrices the notion of gain. This characterization is of great practical value, since good software to compute singular values is widely accessible [10]. Using this singular value concept Doyle proved the following robustness theorem:

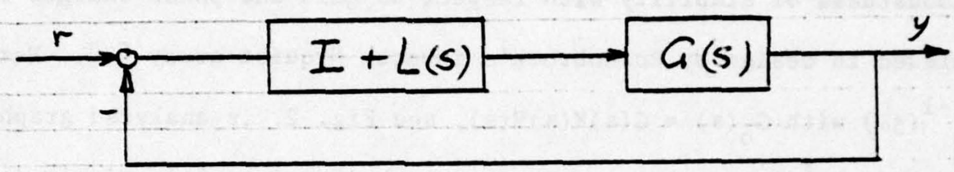

Fig. 3. Perturbation by L(s).

In the system of Fig. 3, let G(s) be rational, square, invertible and such *that the nominal* closed loop with $L(s) = 0$ is stable, i.e. $G(I + G)^{-1} = I + G^{-1}$ is stable. If the system is perturbed by L(s), which by itself is stable, then the perturbed system is stable if

$$\underline{\sigma}(I + G^{-1}(j\omega)) > \overline{\sigma}(L(j\omega)) \quad \text{for all } \omega . \tag{9}$$

For this theorem Sandell [11] gave a different proof, in which G(s) need not be rational. $\underline{\sigma}(I + G^{-1}(j\omega))$ is a frequency dependent measure of robustness in terms of gain margins. For the eigenvalues λ of A (here $= I + G^{-1}(j\omega)$) generally the relation

$$\underline{\sigma}(A) \leq |\lambda(A)| \leq \overline{\sigma}(A) \tag{10}$$

holds. It is possible that the smallest eigenvalue is much larger than $\underline{\sigma}(A)$. Thus the minimum singular value $\underline{\sigma}$ gives a more reliable measure of robustness than the smallest eigenvalue. In fact Doyle constructed an example, where the diagonal dominance approach as well as the characteristic loci approach

generates a Nyquist or Inverse Nyquist plot, which shows $\pm \infty$ db gain margin and 90° phase margin, however the system is only marginally stable.

On the other hand, singular values do not carry phase information, they are real, no Nyquist type encirclement conditions can be obtained.

The problem of uncertainties due to a reduced order design model is interrelated with the question, which modes of the system must be influenced by the control and which others should ideally not be influenced at all. In vehicle control it may for example be desirable to control the rigid body dynamics fast and accurately, i.e. with a reasonably high bandwidth, without interferring with structural vibrations. In frequency domain design techniques, this is achieved by a 40 db/decade roll off beyond the design bandwidth. This aspect is frequently ignored in state space design techniques. In all design techniques it is important to study carefully the behavior in a frequency range above the bandwidth, where modes are still sufficiently controllable and observable, such that the control may move them into the right half s plane.

Stein and Doyle [12] give a design example for a CH-47 helicopter with two control inputs. They apply singular value analysis and the robustness condition (9). Rotor dynamics and rate limits are translated into $\bar{\sigma}(L(j\omega))$ using a result of Safonov [13]. The two singular values were made approximately equal and the bandwidth in both loops was increased as much as $\bar{\sigma}(L(j\omega))$ admitted. A low pass helped to meet the "roll-off" requirement. The example also showed that these methods may lead to very conservative results in cases of large variations of parameters in specific directions, here the flight condition variation.

3.2. State Space Methods

Single-input linear quadratic state feedback regulators have a return difference greater than unity at all frequencies, as was shown by Kalman [14]. Anderson and Moore [15] showed that this fact implies a $\pm 60^{\circ}$ phase margin, infinite gain margin and 50 percent gain reduction tolerance. Safonov and Athans [16] generalized this result to the multiinput case:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ u &= -Kx \end{aligned} \tag{11}$$

with m inputs u_i .

The feedback matrix K is determined by solving a Riccati equation minimizing

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt \tag{12}$$

with Q positive definite and $R = \text{diag}[r_1, \ldots, r_m]$, $r_i > 0$.

The individual inputs u_i are perturbed to $\eta_i u_i$ without interaction between them, i.e.

$$\dot{x} = Ax + B\eta u \quad \text{with } \eta u = \begin{bmatrix} \eta_1 u_1 \\ \vdots \\ \eta_m u_m \end{bmatrix} . \tag{13}$$

Let each perturbation η_i be linear time invariant with proper rational stable transfer function $P_i(s)$. Its frequency response is $P_i(j\omega) = a_i(\omega) \cdot e^{j\phi_i(\omega)}$. Then the closed loop remains stable under a phase perturbation $\phi_i(\omega)$, with $|\phi_i(\omega)| \leq 60^{\circ}$ for all ω. It also remains stable under a gain perturbation $a_i(\omega) \geq 0.5$ for all ω.

Note that this result does not accomodate neglected actuator dynamics for two reasons:

1. Physical actuators have at least 90^{o} phase lag for high frequencies, this can only theoretically be removed by feedback of actuator states, which in turn requires modelling of the actuator as part of eq. (1).

2. The gain and phase margins do not apply simulatneously. It is known from the single input case that gain and phase margins alone may be misleading. The two Nyquist curves in Fig. 4 both have 60^{o} phase margin and infinite gain margin, however $G_2(j\omega)$ has a much smaller distance from the critical point -1 than $G_1(j\omega)$.

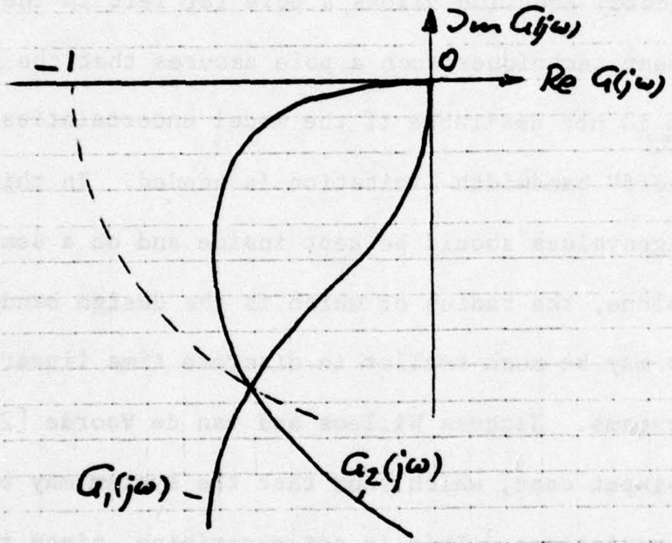

Fig. 4. Two Nyquist curves with 60^{o} phase margin and infinite gain margin.

For this reason Otto Smith [17] used the "complex gain margin," i.e. the minimum distance of $G(j\omega)$ to the critical point, scaled by the local frequency increment along $G(j\omega)$. This approximates the negative real part of a dominant pair of eigenvalues. A multivariable measure for the distance of $G(j\omega)$ from the critical point has been discussed already in form of the singular values of the return difference.

Doyle [18] showed by counterexample that the margins may be arbitrary small, if the state is replaced by a state estimate from a Kalman filter. In his example, the gain margins were arbitrarily small in both the positive and

negative db direction. To improve the margin in this situation, Doyle and Stein [19] developed a "design adjustment procedure," which introduces fictituous noise at the control input to the plant. In this procedure the observer eigenvalues tend to the finite transmission zeros and to infinity. Thus the procedure works only for minimum-phase plants. The procedure is essentially the dual of Kwakernaak's sensitivity recovery method [20]. This however drives the plant poles instead of the observer poles to the transmission zeros, which may lead to large control inputs u.

The solution of the Riccati equation allows a pole far left in the s-plane. Also in pole placement techniques such a pole assures that the system is optimal [15]. This pole is not desirable if the model uncertainties increase with frequency and a "roll-off" bandwidth limitation is needed. In this situation all controlled eigenvalues should be kept inside and on a semicircle in the left half s plane, the radius of which is the design bandwidth.

Gain and phase margins may be much smaller in discrete time linear quadratic state feedback systems. Jacques Willems and van de Voorde [21] give bounds for the single-input case, which show that the system may be very sensitive to feedback gain variations. This is not surprising, since the hold element may be approximated by a phase shift of one half sampling interval.

Safonov and Athans [16] also generalize a single-input result by Anderson and Moore [15], which is useful for actuator nonlinearities. If the perturbation operator η in eq. (13) describes a time varying, memoryless nonlinearity $\eta_i u_i = f_i(u,t)$, then it is a sufficient condition for the closed loop stability, that

$$\frac{1}{2} < \frac{1}{u} f(u,t) \leq M \quad \text{for some } M < \infty \text{ and for all } t . \tag{14}$$

For example for an actuator saturation, stability is guaranteed if the inputs do not exceed twice the saturation level.

Comparisons of numerous optimization techniques for insensitive control systems were made by Harvey and Pope [22,23] for wing load alleviation for the C-5A aircraft and by Vinkler and Wood [24] for a lateral autopilot for a rudderless remotely piloted vehicle. A minimax technique by Salmon [25] and an uncertainty weighting technique by Porter [22] were judged superior to six other techniques in the first report, both however failed in the comparison [27]. Here an expected cost technique by Ly and Cannon [26] and a multistep guaranteed cost technique by Vinkler and Wood [27] came out better than four other techniques. In [23] an information matrix approach by Kleinmann and Rao [28] compared favorably with other techniques.

In problems with insignificant constraints on the control inputs, the weighting matrix R in a quadratic criterion may be small. This leads to high gain solutions as they were discussed in the previous section. A comparison of various high gain feedback systems is made by Young, Kokotović and Utkin [29]. This comparison also includes variable structure systems, which in their sliding mode are insensitive to parameter variations and disturbances, similar to the high-gain system [30]. Young [31] applied this concept to the design of an adaptive model following control system and compared the results for the longitudinal motion of a Convair C-131B aircraft with other model following techniques.

A special case of a high gain control system is useful, if the reference or disturbance input signals can be exactly modelled and asymptotic tracking or disturbance rejection is required. The use of integrators in the loop for zero stationary errors in step and ramp responses is a classical recipe. Also for other inputs an internal model of the input

can be used, e.g. a tuned oscillator (notch filter) for disturbance rejection of helicopter rotor vibrations, whose frequency is regulated. Such a high gain at particular frequencies makes asymptotic tracking robust to plant parameter variations as long as the loop remains stable. This robustness problem was studied by Davison [32] and others. In sampled-data systems the internal model is to be implemented in continuous time, if the tracking property is required also between the sampling instants [33].

Some common problems in all high gain concepts are

- Measurement noise goes highly amplified to the actuator inputs.
- High values for $|u|$ and $|\dot{u}|$ may occur.
- Non-cooperative efforts of the actuators may occur.

The LQG design method offers a systematic way to avoid these difficulties by increase in the R matrix and by the use of a Kalman filter.

4. Robustness With Respect to Large Perturbations in Known Directions.

In the methods of Section 3 relatively little knowledge about the parametric perturbation is assumed. The results are therefore primarily valid for small perturbations. In some cases information is obtained, how big the perturbation is allowed to be in order to maintain stability.

In situations where large perturbations in known directions occur, the previous methods generally lead to very conservative results. In this section some tools are discussed by which such perturbations can be accomodated in the design.

In [34] parameter space methods are used for design. Single-input pole placement is formulated as a linear map from the parameter space $\mathcal{P}$ of coefficients of the characteristic polynomial into the parameter space $\mathcal{K}$ of state feedback gains. It is shown that a characteristic polynomial

$\det(\lambda I - A + bk') = p_0 + p_1\lambda + \dots + p_{n-1}\lambda^{n-1} + \lambda^n$ is assigned to a controllable pair A,b by

$$k' = p'E \tag{15}$$

where

$$p' = [p_0, p_1, \dots, p_{n-1}, 1],\quad E = \begin{bmatrix} e' \\ e'A \\ \vdots \\ e'A^n \end{bmatrix}$$

and e' is the last row of the inverted cotrollability matrix [33].

Example 1: Gain scheduling for the crane of eq. (1) for variable load mass m_i. From eq. (15) $k_1 = p_0 \ell m_c/g$, $k_2 = p_1 \ell m_c/g$, $k_3 = p_0 \ell^2 m_c/g - p_2 \ell m_c + (m_c+m_i)g$, $k_4 = \ell m_c(p_1 \ell/g - p_3)$. The characteristic polynomial remains invariant under large load changes if k_1, k_2 and k_4 are constant as given and k_3 is scheduled by the load according to $k_3 = k_{30} + m_i g$.

Example 2: Small gain stabilization of all cranes by output feedback. The open loop characteristic polynomial $\det(sI-A) = s^2(s^2+\omega^2)$ with four eigenvalues on the imaginary axis is stabilized to $P(s) = \det(sI-A+bk') = (s^2+as+b)(s^2+cs+\omega^2+d)$ with small $a > 0$, $b > 0$, $c > 0$ & small d by $k_1 = (m_c+m_i)b$, $k_2 = (m_c+m_i)a$, $k_3 = \ell(m_i b - m_c d)$, $k_4 = \ell(m_i a - m_c c)$. This describes the cone of stabilizing directions at the origin of the four dimensional $\mathcal{K}$ space. It includes the directions $k_3 = 0$ and $k_4 = 0$, i.e. no feedback of the rope angle and rope angular velocity is necessary if $d = bm_i/m_c$ and $c = am_i/m_c$ are chosen. Thus $k' = [k_1\ k_2\ 0\ 0]$ with small $k_1 > 0$ and $k_2 > 0$ assigns the characteristic polynomial $P(s) = (s^2+as+b)(s^2+(m_i/m_c)as + \omega^2 + bm_i/m_c)$, where $a = k_2/(m_c+m_i)$ and $b = k_1/(m_c+m_i)$. The result is that output feedback $k' = [k_1\ k_2\ 0\ 0]$ stabilizes all cranes with arbitrary positive physical parameters m_c, m_i, ℓ and g.

In the second example of a globally robust system the eigenvalues are moved only incrementally from their widely varying open loop position. This is in agreement with a rule of thumb: If you have to care about constraints on $|u|$ or $|\dot{u}|$, do not try to make a slow system fast or a fast system slow. In other words, under large parameter variations it is not desirable to have only one desired reference model or reference trajectory. Also practically no pilot would expect that an aircraft has the same dynamics in all flight conditions. For the crane it is not necessary to have the same eigenvalues under all loads like in the gain scheduling system of Example 1, it is sufficient to have a minimum damping and minimum negative real part of the eigenvalues for the load range from the empty hook to the maximum load, for which the crane is built. This suggests the idea to specify a region in the eigenvalue plane, in which the eigenvalues shall remain under large parameter variations, instead of a fixed set of eigenvalues. In [34] the boundaries of such regions are mapped into the gain space $\mathcal{K}$. In $\mathcal{K}$ the region is determined in which the feedback gains must be chosen, such that all eigenvalues are in the specified region in the eigenvalue plane. If J pairs A_j, b_j are given (see eq. (2)), then for each pair a region in $\mathcal{K}$ space is obtained. A fixed gain solution does exist if all J regions have a common intersection. This intersection gives an admissible set of feedbacks. A particular element can then be selected under other aspects, e.g. such as to minimize the norm $\sqrt{k'k}$, i.e. the distance from the origin in $\mathcal{K}$ space.

Example 3 [35]: The F4-E aircraft with horizontal canards has a flight envelope as shown in Fig. 5. Four typical flight conditions were chosen for a study. Figure 6 shows the open loop eigenvalues of the longitudial short period mode.

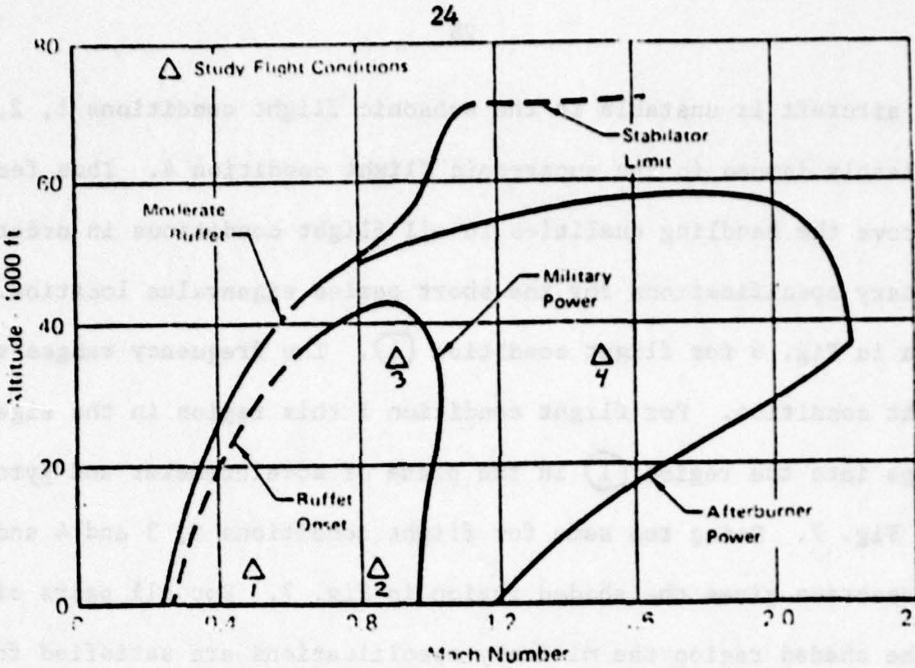

Fig. 5. Four flight conditions of F4-E aircraft.

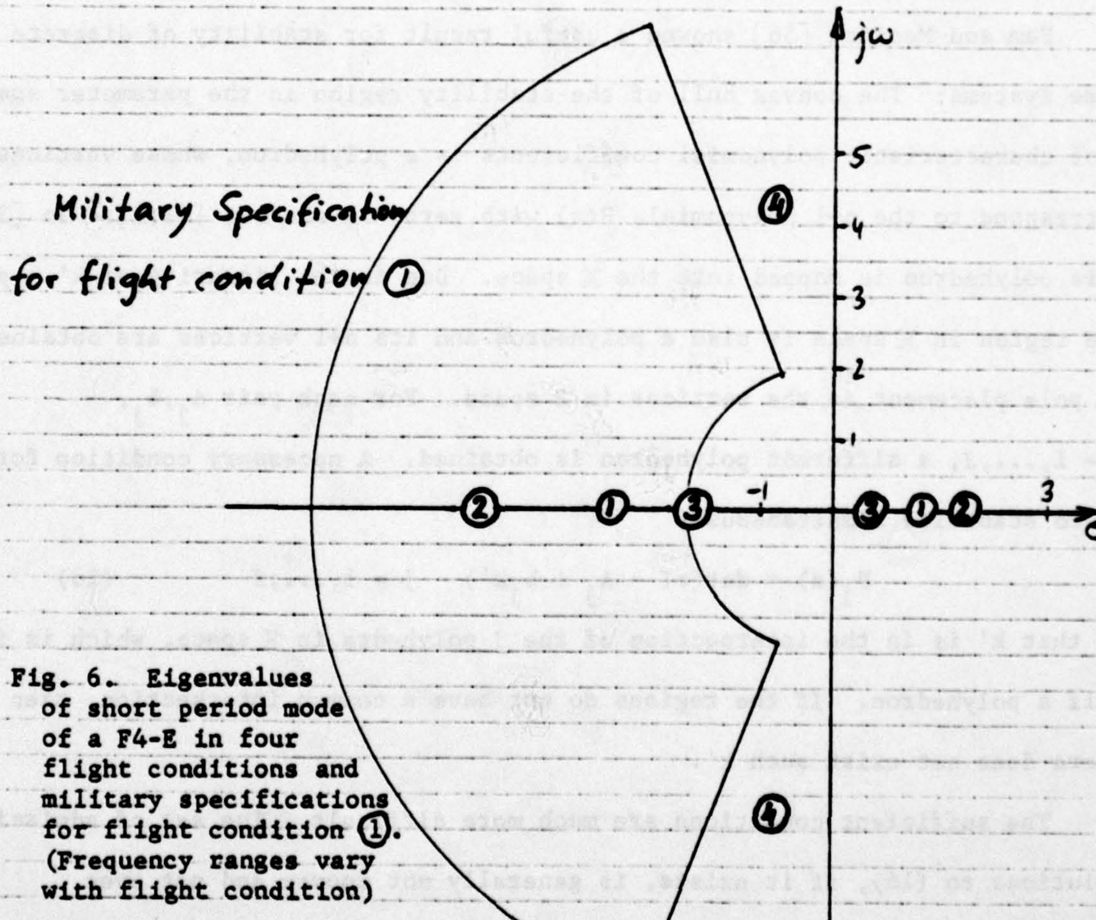

Fig. 6. Eigenvalues of short period mode of a F4-E in four flight conditions and military specifications for flight condition 1. (Frequency ranges vary with flight condition)

The aircraft is unstable in the subsonic flight conditions 1, 2, 3 and insufficiently damped in the supersonic flight condition 4. Thus feedback must improve the handling qualities in all flight conditions in order to meet the military specifications for the short period eigenvalue location. These are shown in Fig. 6 for flight condition 1. The frequency ranges vary with the flight condition. For flight condition 1 this region in the eigenvalue plane maps into the region 1 in the plane of accelerometer and gyro feedback gains in Fig. 7. Doing the same for flight conditions 2, 3 and 4 and taking the intersection gives the shaded region in Fig. 7. For all pairs of gains inside the shaded region the military specifications are satisfied for all flight conditions.

Fam and Meditch [36] showed a useful result for stability of discrete time systems: The convex hull of the stability region in the parameter space $\mathcal{P}$ of characteristic polynomial coefficients is a polyhedron, whose vertices correspond to the n+1 polynomials P(z) with zeros in the set $\{-1,1\}$. In [34] this polyhedron is mapped into the $\mathcal{K}$ space. Due to the linearity of $k' = p'E$ the region in $\mathcal{K}$ space is also a polyhedron and its n+1 vertices are obtained by pole placement ~~in~~ of the vertices in P space. For each pair A_j, b_j, $j = 1,\ldots,J$, a different polyhedron is obtained. A necessary condition for k' to stabilize simultaneously

$$P_j(z) = \det(zI - A_j + b_j k') \quad j = 1,\ldots,J \qquad (16)$$

is that k' is in the intersection of the J polyhedra in $\mathcal{K}$ space, which is itself a polyhedron. If the regions do not have a common intersection, then there does not exist such k'.

The sufficient conditions are much more difficult. The set of admissible solutions to (16), if it exists, is generally not convex and not even connected, such that a search inside the intersecting polyhedron must be made,

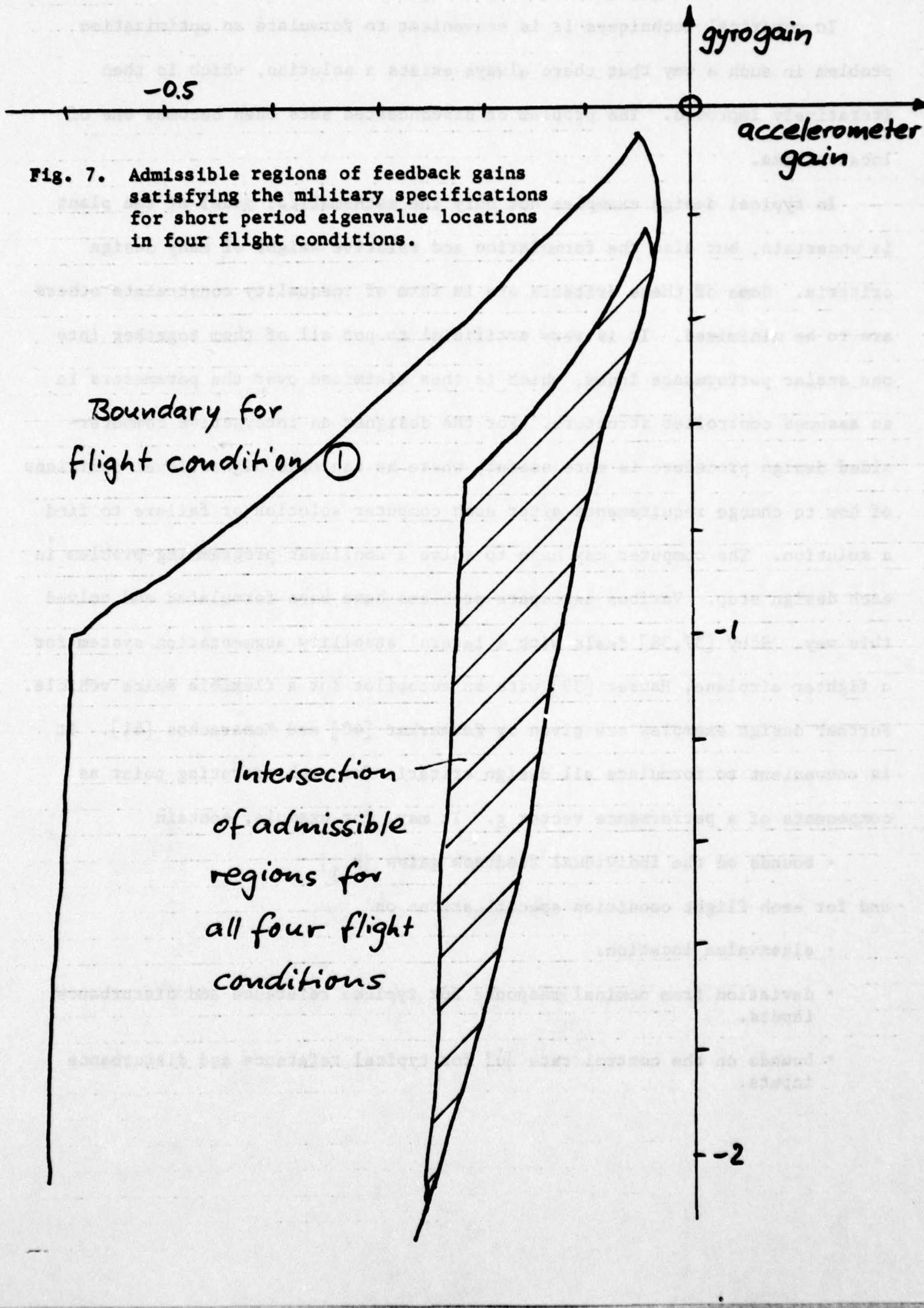

Fig. 7. Admissible regions of feedback gains satisfying the military specifications for short period eigenvalue locations in four flight conditions.

In numerical techniques it is convenient to formulate an optimization problem in such a way that there always exists a solution, which is then iteratively improved. The problem of disconnected sets then becomes one of local minima.

In typical design examples not only the mathematical model of the plant is uncertain, but also the formulation and relative weight of many design criteria. Some of these criteria are in form of inequality constraints others are to be minimized. It is very artificial to put all of them together into one scalar performance index, which is then minimized over the parameters in an assumed controller structure. For the designer an interactive computer-aided design procedure is more useful, where he can make higher level decisions of how to change requirements after each computer solution or failure to find a solution. The computer may have to solve a nonlinear programming problem in each design step. Various aerospace problems have been formulated and solved this way. Schy [37,38] deals with a lateral stability augmentation system for a fighter airplane, Hauser [39] with an autopilot for a flexible space vehicle. Further design examples are given by Karmarkar [40] and Kanarachos [41]. It is convenient to formulate all design criteria for each operating point as components of a performance vector $\underline{g}$. It may, for example, contain

- bounds on the individual feedback gains $|k_{ij}|$

and for each flight condition specifications on

- eigenvalue location.
- deviation from nominal response for typical reference and disturbance inputs.
- bounds on the control rate $|\dot{u}|$ for typical reference and disturbance inputs.

Kreisselmeier and Steinhauser [42] use in an example with five flight conditions of a F4-C aircraft a 40 dimensional vector $\underline{g}$. A vector constraint $\underline{g} \leq \underline{c}$ (i.e. componentwise $g_i \leq c_i$) is given and the feedback gains K are the solution of the problem

$$\underset{K}{\mathrm{Min}}\{\underset{i}{\mathrm{Max}}\ g_i(K)/c_i\} . \qquad (17)$$

Using an algorithm described in [43] Kreisselmeier and Steinhauser obtain a Pareto-optimal solution. Figure 8 shows some reference step responses of this design for an F4-C. It is stable in the five flight conditions. The open loop responses on the left side show that the aircraft is slow in flight condition 1 (landing approach). Here a slower reference response was given than for the high speed conditions 2 and 4. The desired reference response was specified as $g_i(t) = g_M(\alpha_i t)$ where for each flight condition $i = 1,2,\ldots,5$ an appropriate time scale α_i was chosen. This resulted in the insensitive closed loop responses on the right side of Fig. 8, which required only a relatively small control rate $|\dot{u}|$. The same feedback resulted in similarly good disturbance responses.

Also the results of Shy [38] showed that an amazingly large variation of parameters can be accomodated by a fixed gain controller, if the requirements were in good agreement with the physical limitations. These designs result in low gain solutions, and the dynamics change in an acceptable or desirable way as the physical parameters vary.

5. Integrity, Robustness With Respect to Sensor and Actuator Failures

If an actuator or sensor is connected to a high gain, then its failure is a larger perturbation than in a low gain situation. Thus requirements for robustness with respect to actuator and sensor failures tend to result in low gain solutions. Even more important is the aspect of avoiding non-cooperative

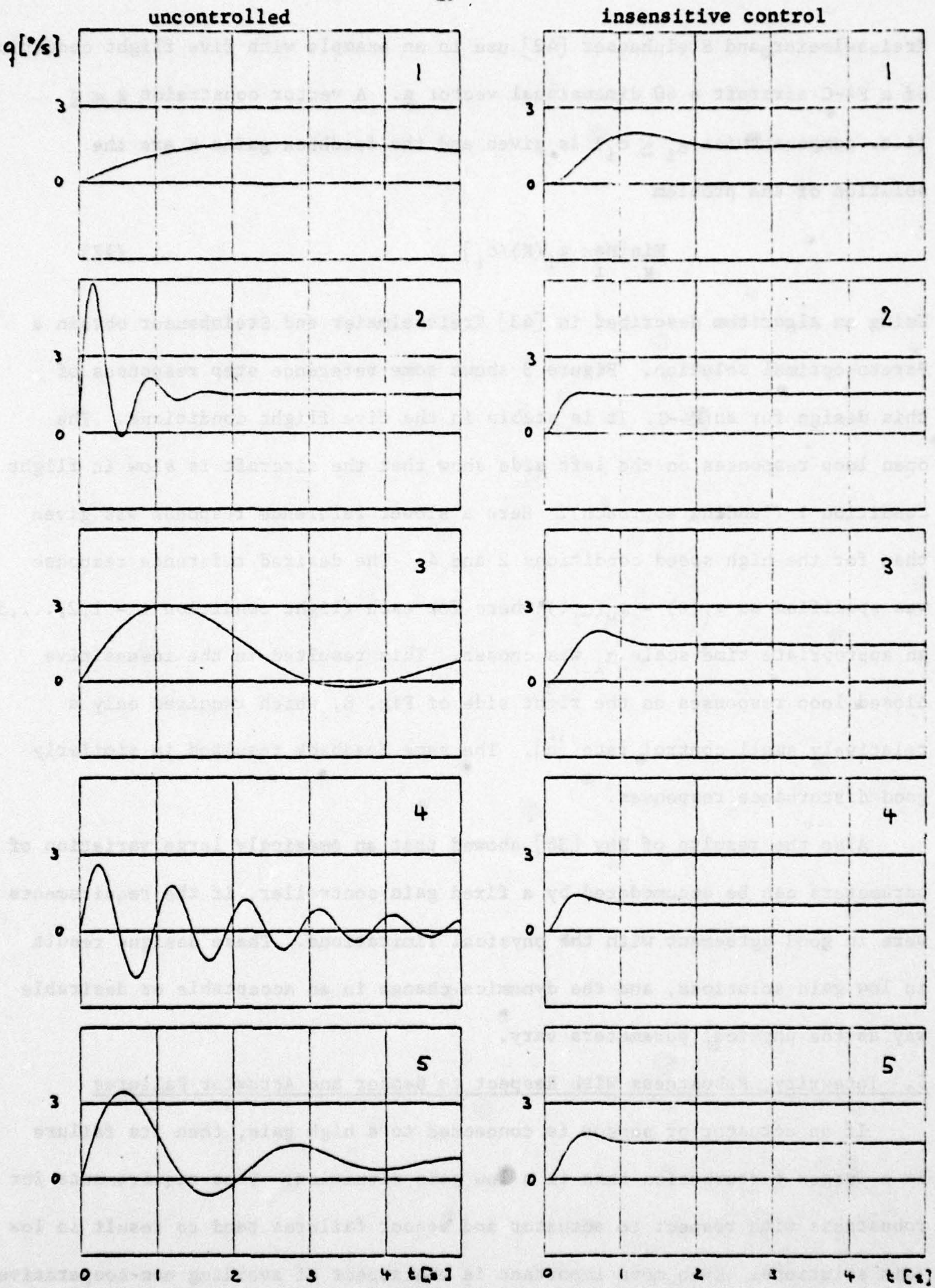

Fig. 8. Response to a step command. F-4 (Phantom) aircraft at 5 extremal flight conditions (altitude 0 ... 40 000 ft. Mach number .2 ... 2.2).

efforts of actuators. If, for example, one input alone places some eigenvalues in the right half plane and another one is needed to bring them back into the left half plane, then apparently no robustness of stability with respect to actuator failures can be achieved.

One approach to achieve robustness of stability with respect to certain failures is to try to extend gain reduction margins to include gain zero. Belletrutti and MacFarlane [44] use the term "high integrity" for robustness with respect to certain failures. They check the stability conditions for gains reduced to a small ε using Nyquist stability criteria for characteristic loci of principal submatrices of the return ratio. In this analysis the loop must be broken at the point where the actual failure may occur and thus the gain reduction margin is needed. Owens [45] derived necessary and sufficient conditions for integrity of systems with multivariable proportional-integral controllers.

Solheim [46] formulated the integrity problem in the context of quadratic optimal control. In examples an increased integrity is obtained with an increased weight R on the control in the quadratic criterion, another indication that the solution will tend to a low gain solution. Wong, Stein and Athans [47] show the following gain reduction result for LQ regulators: The matrix $A_c(\Lambda) = A + B\Lambda K$ with $\Lambda = \mathrm{diag}[\alpha_1 \ldots \alpha_m]$, where K minimizes $\int_0^\infty x'Qx + u'Rx dt$ for $\Lambda = I$, is stable for all

$$\Lambda > \frac{1}{2}\,[I - (R^{1/2}\,K'Q^{-1}KR^{1/2})^{-1}]\ . \tag{18}$$

This generalizes the bound $\alpha_i > 0.5$ from [16]. The recommendation is, from a purely robustness standpoint, to choose Q and R such as to maximize

$$\lambda_0 = \lambda_{min}\{(R^{1/2}K'Q^{-1}KR^{1/2})^{-1}\}\ . \tag{19}$$

Kreisselmeier [48] proposes to modify the quadratic criterion, where for each considered failure situation, a quadratic criterion is formulated and the overall criterion is a weighted sum of these terms.

In failure situations it may be desirable to specify other emergency boundaries in the eigenvalue plane than only the imaginary axis. This problem is treated by parameter space methods in [34]. The concept is illustrated for the case of sensor failures in Fig. 9. A nominal region for the eigenvalue location and a larger emergency region are mapped into the space of

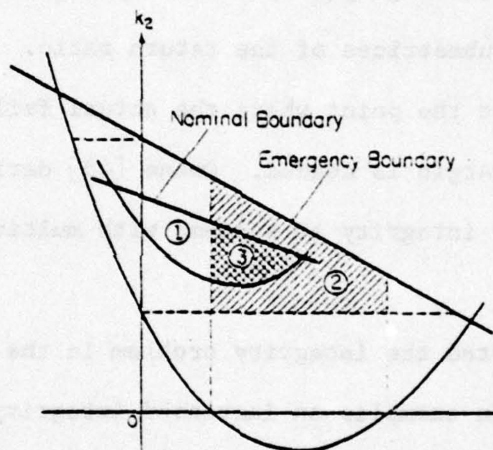

Fig. 9. Illustration of failure robustness and emergency boundaries.

feedback gains. It is assumed that the system is represented in "sensor coordinates," then a failure of a sensor for state variable x_i corresponds to switching k_i to zero. The projection of point 1 on the k_1 axis is outside the emergency boundary, i.e. the emergency specification is not robust with respect to a sensor failure $k_2 = 0$. It is, however, robust with respect to $k_1 = 0$. For all points in the shaded area the emergency specifications are robust with respect to either sensor failure. An alternative to this robust solution would be in this example to omit sensors 1 and to use multiplexed

sensors for x_2 and failure detection.

In the multiinput case a sensor failure is equivalent to changing a column of the K matrix to zero and an actuator failure is equivalent to changing a row of K to zero. In [34] an actuator failure example is studied, where the problem is formulated such that the eigenvalues are placed in a nominal position with two actuators and move as little as possible towards the stability boundary for failures of either one of two actuators.

Apparently a necessary condition for robustness with respect to failures is that the insufficiently damped eigenvalues (outside the specified region) remain controllable and observable after the failure. In the crane example, the sensor for the crab position x_1 is essential, because x_1 is not observable by other states. In such situations it is apparently misleading to use high gain feedback and to show gain reduction to only a few percent of the high gain. For failures of essential actuators and sensors only redundant components can help.

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MODEL REFERENCE ADAPTIVE CONTROL AND STOCHASTIC SELF-TUNING REGULATORS - TOWARDS CROSS-FERTILIZATION

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Abstract

Model Reference Adaptive Systems (MRAC) where the control objectives are specified by a reference model and Stochastic Self-Tuning Regulators (S-STURE) where the control objectives are specified by an A.R.M.A. model are presented together from a duality point of view. The duality existing between these two classes of adaptive systems extends the duality existing in the linear case with known parameters between minimum variance control and modal control.

The equivalence between Implicit and Explicit M.R.A.C. is discussed and the corresponding S-STURE with explicit and implicit prediction reference models are defined. Tools for analysis of M.R.A.C. and S-STURE in a deterministic and a stochastic environment respectively are given. They are also used together with the duality aspects to explore the behaviour of M.R.A.C. in a stochastic environment and the behaviour of S-STURE in a deterministic environment.

Finally, the design of schemes behaving as a desired M.R.A.C. in a deterministic environment and as a desired S-STURE in a stochastic environment is indicated and an example is given.

1. INTRODUCTION

Recent works [1], [2], [3] have enlighted the connections existing between adaptive control systems with an explicit reference model (called also direct adaptive control [2]) where the parameters of the controller are directly adapted and the adaptive control systems with an implicit reference model

(called also indirect adaptive control [2]) where an adaptive predictor derived from M.R.A.S. techniques is used as an intermediate step and the parameters of the adaptive predictor are used to up-date the controller. These two schemes designed from a stability point of view, to operate in a deterministic environment can be equivalent. The first condition which should be satisfied in order that the two schemes be equivalent is that the control strategy for the implicit M.R.A.C. is such that the output of the adaptive predictor behaves identically to that of the explicit reference model (i.e. the adaptive predictor plus the controller form and implicit reference model).

In refs. [1], [4], connections between M.R.A.C. designed to operate in a deterministic environment and the Minimum Variance-Self Tuning Regulator (MV-STURE) designed to operate in a stochastic environment have been investigated (the MV-STURE has a structure similar to the implicit M.R.A.C.). Furthermore as it will be shown in this paper, minimum variance control and respectively MV-STURE appear as particular cases of stochastic control and stochastic STURE where the control objectives are specified by an ARMA model.

Therefore, the basis for a unified approach to M.R.A.C. and S-STURE exist. Work in this direction have been done in [5] where the on-line parameter estimation via prediction error methods have been emphasized as the common interpretation of the various schemes. Another work towards a unified approach to M.R.A.C. and S-STURE on which the present paper is largely based is presented in [6], where the duality between linear deterministic modal control and minimum variance stochastic control has been extended to M.R.A.C. and S-STURE and the interpretation of S-STURE as "stochastic" M.R.A.C. has been sketched.

In the present paper, the ideas from [6] are developed; structural similarities duality and equivalence as well as the differences between various schemes are more deeply investigated. This allows:

1) To analyse in a straightforward way the behaviour of M.R.A.C. in a stochastic environment and vice-versa the behaviour of S-STURE in a deterministic environment.

2) To define new schemes of M.R.A.C. and S-STURE which can offer better performances in some situations.

3) To design adaptive control schemes which can operate in a deterministic environment as a desired M.R.A.C. and in a stochastic environment as a S-STURE.

The common denominators for this unified approach for M.R.A.C. and S-STURE are the structure of the parameter adaptation algorithm and the presence of a reference model (implicit or explicit) in the deterministic environment and its counterpart the prediction reference model (implicit or explicit) in the stochastic environment.

The paper is organized as follows. Section 2 recalls the duality between minimum variance control and stochastic control. In Section 3, this duality is extended for the case of Linear Model Following Control where the design objectives are specified by a difference equation and a class of linear stochastic controllers where the design objectives are specified by an ARMA model. In Section 4, the principles and block diagrams of Explicit M.R.A.C., Implicit M.R.A.C. and S-STURE, which are extensions of the linear control strategies discussed in Section 3 when the plant parameters are unknown, are reviewed and the structural similarities are emphasized. In Section 5, tools for analysis of the various schemes in a deterministic environment and a stochastic environment are given. These tools are based on the EFR method (Equivalent Feedback Representation) and the ODE method (Ordinary Differential Equation) respectively. In Section 6, a MV-STURE scheme and an Implicit M.R.A.C. scheme are presented and they will serve as a basis for illustrating the properties of various configurations. In Section 7, the equivalence between

implicit and explicit M.R.A.C. is discussed. In Section 8, the asymptotic duality between M.R.A.C. and S-STURE is examined (and M.R.A.C. dual to MV-STURE are constructed). In Section 9, the interpretation of S-STURE as stochastic M.R.A.C. using an implicit reference prediction model and a new equivalent realization of S-STURE using an explicit reference prediction model are given. In Section 10, the problem of the positivity conditions required for the convergence of both M.R.A.C. and S-STURE is examined in connection with a converse dual problem. In Sections 11 and 12, the behaviour of MRAC in a stochastic environment and of S-STURE in a deterministic environment is examined in connection with the duality MRAC-S-STURE. A combined MRAC-S-STURE scheme is then derived in Section 13. This scheme can operate in a deterministic and stochastic environment as a MRAC and a S-STURE respectively.

In order to reduce the technical details and to make the presentation more transparent, throughout the paper the plant to be controlled in the absence of disturbances is assumed to be described by a discrete rational transfer function with a basic delay of one sample and with the leading coefficient of the numerator being known and constant. All the results can be extended for the case of a general delay and unknown numerator leading coefficient and this will be the object of a forthcoming paper.

2. THE DUALITY BETWEEN MINIMUM VARIANCE CONTROL AND THE MODAL CONTROL

2.1 Stochastic case

Consider the process to be controlled and its stochastic environment described by:

$$y_k = \frac{B(q^{-1})}{A(q^{-1})} u_{k-1} + \frac{C(q^{-1})}{A(q^{-1})} v_k = \sum_{i=1}^{n} a_i y_{k-1} + \sum_{i=1}^{m} b_i u_{k-i-1}$$

$$+ b_o u_{k-1} - \sum_{i=1}^{n} c_i v_{k-i} + v_k = P_0^T \phi_{k-1} + b_o u_{k-1} + C(q^{-1}) v_k \quad (2.1)$$

where y_k is the measured output, u_k is the control and v_k is a sequence of equally distributed independent normal $(0, \sigma)$ random variables and:

$$A(q^{-1}) = 1 - a_1 q^{-1} \dots - a_n q^{-n} \tag{2.2}$$

$$B(q^{-1}) = b_o + b_1 q^{-1} - \dots + b_m q^{-m} \tag{2.3}$$

$$C(q^{-1}) = 1 - c_1 q^{-1} \dots - c_n q^{-n} \tag{2.4}$$

$$P_0^T = [a_1 \dots a_n, b_1 \dots b_m] \tag{2.5}$$

$$\phi_{k-1}^T = [y_{k-1} \dots y_{k-n}, u_{k-2} \dots u_{k-m-1}] \tag{2.6}$$

The polynomial $B(z^{-1})$ and $C(z^{-1})$ are supposed to have all zeros in $|z| < 1$.

The minimum variance control is calculated such that the following objectives are met: 1) $E\{y_k\} = 0$, 2) $E\{y_k^2\} = \min$. The minimum variance control can be calculated directly [7] and one gets:

$$u_{k-1} = -\frac{1}{b_o}[P_{MV}^T \phi_{k-1}] \tag{2.7}$$

where:

$$P_{MV}^T = a_1 - c_1 \dots a_n - c_n, b_1 \dots b_n \tag{2.8}$$

With this control, $E\{y_k^2\} = E\{v_k^2\}$ and $\lim_{k \to \infty} y_k = v_k$.

For the better understanding of the minimum variance self-tuning regulator and its connections with M.R.A.S., it is worth to recall that the minimum variance control can be obtained using the separation theorem, i.e.:

1) Design an optimal predictor $\hat{y}_{k/k-1}$ from (2.1).

$$\hat{y}_{k/k-1} = P_0^T \phi_{k-1} + b_0 u_{k-1} - \sum_{i=1}^{n} c_i v_{k-i} \tag{2.9}$$

2) Determine a control for the predictor (2.7) in order to achieve the deterministic objective $\hat{y}_{k/k-1} \equiv 0$ (and take advantage that in this case $y_k = v_k$).

This strategy to compute the MV control will then be extended in order to obtain a MV-STURE when the plant parameters and disturbance parameters are unknown.

Note also that the minimum variance control strategy can be formulated in a different way which will be exploited later: for the process and the stochastic disturbance given in (2.1), find u_k such that $\lim_{k \to \infty} y_k = v_k$.

2.2 Deterministic case

The process to be controlled is described by:

$$y_k = P_0^T \phi_{k-1} + b_0 u_{k-1} \quad ; \quad y(0) \neq 0 \tag{2.10}$$

where P_0 and ϕ_{k-1} are given by (2.5) and (2.6).

The objective of the "modal control" is either to find u_k such that $y_k \equiv 0$, $\forall\, k \geq 1$ (all the closed loop poles are at the origin) or such that:

$$A^0(q^{-1}) y_k = 0 \tag{2.11}$$

where:

$$A^0(q^{-1}) = 1 - a_1^0 q^{-1} \;\ldots\; - a_n^0 q^{-n} \tag{2.12}$$

defines the desired poles of the closed loop system. A simple analysis shows that the modal control is:

$$u_{k-1} = -\frac{1}{b_0}\,[P_{MC}^T \phi_{k-1}] \tag{2.13}$$

where

$$P_{MC} = [a_1 - a_1^0, \;\ldots\; a_n - a_n^0, \; b_1 \;\ldots\; b_n] \,. \tag{2.14}$$

If now $A^0(q^{-1}) = C(q^{-1})$ where $C(q^{-1})$ defines the stochastic disturbance in (2.1), the controls given by (2.7) and (2.13) are the same. One has therefore the following result.

THEOREM 2.1: (Duality between minimum variance control and modal control):

The minimum variance control of the process and its stochastic environment (2.1) is identical to the modal control for the same process in a deterministic environment if and only if the desired closed loop behaviour is defined by:

$$C(q^{-1})y_k = 0 \tag{2.15}$$

3. LINEAR MODEL FOLLOWING CONTROL AND LINEAR STOCHASTIC (ARMA) MODEL FOLLOWING CONTROL

The remarks made in Section 2 can be generalized as it will be shown next.

3.1 Deterministic case

In the case of a deterministic environment not only the regulation objectives are specified but also the tracking objectives are specified. For a deterministic environment, the objectives are defined as follows:

Regulation

For the plant given by (2.8) find the control u_k such that:

$$A^0(q^{-1})y_k = 0 \, . \tag{3.1}$$

Tracking

For the plant given by (2.8), find the control u_k such that:

$$A^0(q^{-1})y_k = B^0(q^{-1})u_k^R \tag{3.2}$$

where:

$$B^0(q^{-1}) = b_1^0 q^{-1} + b_2^0 q^{-2} + \dots b_m^0 q^{-m} \tag{3.3}$$

and u_k^R is the reference input.

Note that the polynomials $A^0(q^{-1})$ are not necessarily the same in Eq. (3.1) and Eq. (3.2).

The control objectives (3.1) and (3.2) can be specified explicitly using an appropriate "reference model" of parallel, series or series-parallel structure [8] and imposing that the plant-model error goes to zero. One obtains in this way a linear model following control system.

This approach will then allow to construct MRAC when the plant parameters are unknown or vary during operation. Examples of linear model following control schemes which allow to achieve the control objectives (3.1) or (3.2) will be given next (other configurations are also possible).

Regulation

Consider the plant given by (2.10) and the regulation objective (3.1). Define the "series" reference model:

$$x_k = \hat{y}^M_{k/k-1} = \left(\sum_{i=1}^{n} a_i^0 q^{-i}\right) y_k \ . \tag{3.4}$$

(The output of the reference model can be interpreted as the desired predicted value based on the measurements up to k-1.)

The regulation objective (3.1) will be achieved if instead of (3.1) one considers the new objective:

$$y_k - x_k = y_k - \hat{y}^M_{k/k-1} = 0 \tag{3.5}$$

and the resulting control is:

$$u_{k-1} = -\frac{1}{b_0}\left[P^T_{MC}\phi_{k-1}\right] \tag{3.6}$$

where:

$$P^T_{MC} = \left[a_1 - a_1^0 \ \ldots \ a_n - a_n^0,\ b_1 \ \ldots \ b_m\right] \ . \tag{3.7}$$

Tracking

Consider a "parallel" reference model whose output specifies the tracking objective:

$$x_k = (\sum_{i=1}^{n} a_i^0 q^{-i})x_k + (\sum_{i=1}^{m} b_i^0 q^{-i})u_k^R . \tag{3.8}$$

The design objective (3.2) can be replaced by the objective:

$$y_k - x_k = 0 \tag{3.9}$$

which leads to the control:

$$u_{k-1} = -\frac{1}{b_0}[P_0^T \phi_{k-1} - (\sum_{i=1}^{n} a_i^0 q^{-i})x_k - (\sum_{i=1}^{m} b_i^0 q^{-i})u_k^R] . \tag{3.10}$$

But, instead of the design objective (3.9), one can consider the following one:

$$A^0(q^{-1})(y_k - x_k) = 0 \tag{3.11}$$

which leads to the control:

$$u_{k-1} = -\frac{1}{b_0}[P_{MC}^T \phi_{k-1} - (\sum_{i=1}^{m} b_i^0 q^{-i})u_k^R] . \tag{3.12}$$

Note that the "parallel" reference model (3.8) with the control objective (3.11) can be replaced by a "series-parallel" reference model of the form:

$$x_k = (\sum_{i=1}^{n} a_i^0 q^{-i})y_k + (\sum_{i=1}^{m} b_i^0 q^{-i})u_k^R$$

with the objective (3.9) and the corresponding control will be given again by (3.12).

3.2 Stochastic case

The minimum variance control can be interpreted in two ways: 1) it is the solution for a particular case of L.Q.G. problem; 2) it is the solution in order to obtain $y_k = v_k$ which is a particular A.R.M.A. model.

This second interpretation allows to consider as a generalization of the minimum variance control, the class of linear stochastic controls which in a given stochastic environment achieve a control objective specified by an ARMA process. The control objectives are specified in the following way.

Regulation

For the plant and its stochastic environment given by (2.1), find a control u_k such that:

$$D(q^{-1})y_k = F(q^{-1})v_k \tag{3.13}$$

where:

$$D(q^{-1}) = 1-d_1q^{-1} - \dots - d_nq^{-n} \tag{3.14}$$

$$F(q^{-1}) = 1+f_1q^{-1} + \dots + f_rq^{-r} \tag{3.15}$$

and v_k is the sequence of independent random normal variables $(0, \sigma)$ generating the disturbance in (2.1).

One recognizes that minimum variance control corresponds to $D(q^{-1}) = 1$, $F(q^{-1}) = 1$ when the delay is 1 as in Eq. (2.1) and to $D(q^{-1}) = 1$ and $F(q^{-1}) = (1 + \sum_{i=1}^{r} f_iq^{-i})$ when the delay is r+1 [7].

Tracking

For a plant given by:

$$y_k = P_0^T\phi_{k-1} + b_0u_{k-1} \tag{3.16}$$

and an A.R.M.A. process given by:

$$A^0(q^{-1})x_k = B^*(q^{-1})v_k \tag{3.17}$$

where

$$B^*(q^{-1}) = 1 + B^0(q^{-1}) \tag{3.18}$$

and v_k is a sequence of independent random normal variables $(0, \sigma)$, find a control u_k such that:

$$D(q^{-1})(x_k - y_k) = F(q^{-1})v_k \,. \tag{3.19}$$

Similar to the deterministic case, the objectives can be specified using "prediction reference models" and imposing an appropriate behaviour to the plant-model error. This scheme will be called "Linear Stochastic (ARMA) Model Following Control" systems. Examples of such configurations will be given next.

<u>Regulation</u>

For the plant and its stochastic environment (2.1), assume that the following control objective should be achieved:

$$D(q^{-1})y_k = v_k \,. \tag{3.20}$$

Define a "series" reference prediction model:

$$\hat{y}^M_{k/k-1} = x_{k/k-1} = (\sum_{i=1}^{n} d_i q^{-i})y_k \tag{3.21}$$

then, the regulation objective (3.20) will be achieved if one considers the new objective:

$$y_k - \hat{y}^M_{k/k-1} = v_k \,. \tag{3.22}$$

This can be obtained with:

$$u_{k-1} = -\frac{1}{b_0}\{[\sum_{i=1}^{n}(a_i - d_i)q^{-i}]y_k + (\sum_{i=1}^{m} b_i q^{-i})u_{k-i-1}$$

$$- (\sum_{i=1}^{n} c_i q^{-1})(y_k - \hat{y}^M_{k/k-1})\} \,. \tag{3.23}$$

One recognizes easily that for $d_i = 0$, $i = 1,2,\ldots,n$, $\hat{y}^M_{k/k-1} \equiv 0$ and one obtains the minimum variance control for the plant described by (2.1). Note

also that the control (3.23) can be interpreted as one which minimizes the variance of the plant-model error (min $E\{(y_k-\hat{y}^M_{k/k-1})^2\}$).

Tracking

The reference prediction model is given directly by the ARMA process which should be tracked. Consider the following two cases:

a) $D(q^{-1}) = F(q^{-1}) = 1.$

In this case, u_k should be such that:

$$x_k - y_k = v_k \tag{3.24}$$

which in fact corresponds to min $E\{(x_k-y_k)^2\}$. u_k will be computed such that y_k be the best predictor in the mean square of x_k. But, from [7], it is known that the optimal predictor for x_k will be:

$$\hat{x}_{k/k-1} = (\sum_{i=1}^{n} a_i^0 q^{-i})x_k + (\sum_{i=1}^{m} b_i^0 q^{-i})v_k = y_k . \tag{3.25}$$

From Eqs. (3.16) and (3.25), one obtains:

$$u_{k-1} = -\frac{1}{b_0}[P^T_{MC}\phi_{k-1} - (\sum_{i=1}^{n} a_i^0 q^{-i})(x_k-y_k)$$

$$-(\sum_{i=1}^{m} b_i^0 q^{-i})(x_k-y_k)] = -\frac{1}{b_0}[P_0^T\phi_{k-1}$$

$$-(\sum_{i=1}^{n} a_i^0 q^{-i})x_k - (\sum_{i=1}^{m} b_0^i q^{-i})(x_k-y_k)] \tag{3.26}$$

where P_{MC} is given by Eq. (3.7).

b) $D(q^{-1}) = A^0(q^{-1})$; $F(q^{-1}) = 1$

In this case, one obtains:

$$u_{k-1} = -\frac{1}{b_0}\{P^T_{MC}\phi_{k-1} - (\sum_{i=1}^{m} b_i^0 q^{-i})[A^0(q^{-1})(x_k-y_k)]\} \tag{3.27}$$

by taking in account that $A^0(q^{-1})(x_k-y_k) = v_k$.

3.3 Duality

The duality holds also between Linear Model Following Control and Linear Stochastic (ARMA) Model Following Control.

For regulation, comparing Eq. (3.6) and Eq. (3.23), one concludes that duality holds either for:

$$c_i = 0 \quad ; \quad d_i = a_i^0 \tag{3.28}$$

or

$$d_i = 0 \quad ; \quad c_i = a_i^0 . \tag{3.29}$$

For tracking comparing Eq. (3.10) with Eq. (3.26), and Eq. (3.12) with Eq. (3.27), duality holds if v_k is replaced by u_k^R in the deterministic case.

4. EXPLICIT AND IMPLICIT M.R.A.C. AND STOCHASTIC S.T.U.R.E.

When the parameters of the plant (and those of the disturbance in the stochastic case) are unknown or vary during operation and adaptive approach should be considered in order to asymptotically achieve the design objectives.

The connections existing between the techniques indicated in the title of the paragraph have been emphasized to a certain extent only in the last years and the reason is that their original development have been done from different points of view. The M.R.A.C. techniques have been initially developed for deterministic continuous time tracking problems and the minimum variance STURE (which is a particular stochastic STURE) has been initially developed for stochastic discrete time regulation problems.

We will examine next some structural similarities between the various schemes which in fact are partly behind the connections which can be established between them.

A basic scheme for an explicit M.R.A.C. is given in Fig. 4.1. In this scheme, a reference model specifies the objectives either for tracking or for

regulation (for details, see [8] and [9]) and the controller is directly adapted by the adaptation mechanism which process the plant-model error.

A basic scheme for implicit M.R.A.C. is given in Fig. 4.2. An adaptive predictor derived from MRAS techniques is used as an intermediate step and the parameter of the predictor is used for updating the controller such that the controller plus the adapative predictor behaves like a reference model (i.e. they form an implicit reference model). It is clear that a first condition in order that the two schemes be equivalent is that the two reference models be the same and this is illustrated in Fig. 4.3.

In this case, the two schemes can be equivalent if the error between the plant and the explicit reference model for the scheme of Fig. 4.1 and the error between the plant and the adaptive predictor (the output of the implicit reference model) in Fig. 4.2 will behave identically. Explicit conditions for this will be given in Section 7.

It is also interesting to remark that the scheme of Fig. 4.2 uses an extension of the separation theorem, i.e. in a first step, one designs a predictor of the controlled output and in the second step, one designs a control for this predictor such that the predictor output satisfies the design objectives. The error with respect to the initial objective being the prediction error.

The basic scheme illustrating the "STURE" philosophy is given in Fig. 4.4. One recognizes two steps which are inter-related, i.e. 1) on-line parameter estimation; 2) computation of the controller from the current parameter estimates. These two steps are inter-related because in many cases by a convenient parameterization of the prediction model used for parameter estimation, the computation of the controlled is drastically simplified. This simplification arises when the controller parameters can be explicitly expressed by linear relations

in terms of parameter estimates. One says that the prediction model is parameterized in terms of controller parameters. Aström and co-workers call this a STURE with "implicit" identification. For details, see [10].

However, for a better understanding of the STURE, one should recall that all the on-line parameter estimation techniques are prediction error methods, i.e. they use an adaptive predictor which is updated by an on-line identification algorithm (adaptation mechanism). With this remark and for the case when the controller parameters depend explicitly and linearly on the estimated ones, the STURE have always the structure shown in Fig. 4.5.

One immediately recognizes a structural similarity with the implicit MRAC shown in Fig. 4.2. The similarity goes further if we note that in the linear case with constant parameters, the M.V. control (see Section 2) can be obtained in two steps:

1) Optimal predictor

2) Control of the predictor in order to achieve a deterministic objective.

The output of the predictor behaves in this case as an implicit reference prediction model. This strategy is then extended for MV-STURE with the difference that the optimal linear predictor is replaced by an adaptive predictor. In this case, the controller plus the adaptive predictor form an implicit reference prediction model.

The linear stochastic (ARMA) model following control systems when the plant parameters are unknown leads also to STURE configurations of the form given in Fig. 4.5, but as it will be shown in Section 9, another configuration is also possible.

Few comments should be made upon the terms "self-tuning" and "adaptive" which are directly connected with the nature of the adaptation gains. In the "self-tuning" case, one assumes that the plant parameters are constant but

unknown, which together with the presence of the stochastic environment, leads as for on-line identification of linear plants with constant parameters to the use of time-decreasing adaptation gains (least squares types algorithms or stochastic approximation types algorithms). However in order to be able to track slowly time varying plants, algorithms with time varying adaptation gains have been introduced. An analysis of the behaviour of the S-STURE becomes much more complicated in this case. In the "adaptive" case, one assumes that the environment is deterministic and therefore asymptotic stability of the full system can be assured using either constant, time-decreasing or time-varying adaptation gains. It is assumed also that plant parameters are constant over large periods of time, but when using constant or appropriate time-varying adaptation gain, the adaptive system can react to a change in parameters occurring at a random instant which is not the case when using time-decreasing adaptation gain.

However when M.R.A.C. are used in a stochastic environment convergence to fixed values of controller parameters can be obtained (and analysed) only when time-decreasing adaptation gains are used.

Note also that time-decreasing and time-varying adaptation gains provide better adaptation performance than constant adaptation gains even in a deterministic environment since they modify not only the magnitude of the correction at each step but also the direction.

5. TOOLS FOR ANALYSIS AND SYNTHESIS

Both M.R.A.C. and stochastic STURE are non-linear time-varying systems and, therefore, one of the crucial points is to examine their convergence properties as time goes to infinity. This immediately brings the stability aspects in view. In fact, the analysis and synthesis of both M.R.A.C. and STURE

involve two points, one of algebraic nature: does a linear controller which can achieve the objectives exist? And this is equivalent to defining the desired equilibrium point of the system, and the second point involves a stability analysis with respect to this possible equilibrium point.

However since M.R.A.C. are of deterministic nature and S-STURE are of stochastic nature the stability concepts and tools of analysis will of course be different. Note also that once stability concepts and appropriate tools for analysis being defined the problem can be reversed into a design one by imposing to find an adaptation mechanism which assures that the equilibrium point have the desired stability properties.

The two methods which will be presented next, one for analysis of the global asymptotic stability in a deterministic environment and the other one for the analysis of the w.p.1 convergence in a stochastic environment have been successfully used for analysis and design of on-line identifiers, adaptive observers and adaptive state estimators [11], [12], [13]. However when using them for the analysis and design of MRAC and S-STURE, they cannot give a full proof for global convergence since the proof of boundness of some variables (control and plant output) requires an additional analysis [5], [14]. Nevertheless recent work has shown that adaptation mechanisms designed using these approaches assure also the boundness of the various variables which imply that global convergence is assured [5], [14], [15]. In order to carry on the similarity between MRAC and S-STURE the adaptation algorithm used for MRAC will be particularized to the form which is identical to that used for S-STURE (for other adaptation algorithms, see [9], [16]).

The basic parametric adaptation algorithm considered throughout the paper is:

$$\hat{\beta}(k) = \hat{\beta}(k-1) + F_{k-1}\phi_{k-1}\nu_k \qquad (5.1)$$

where ν_k is in general a linear combination of the "a posteriori" generalized error ϵ_k (called also "a posteriori" plant-model error) and of its previous values $\epsilon_{k-1}, \dots \epsilon_{k-n}$, ϕ_{k-1} is the observation vector and F_{k-1} is the adaptation gain given by:

$$F_k^{-1} = \lambda_1(k)F_{k-1}^{-1} + \lambda_2(k)\phi_{k-1}\phi_{k-1}^T \tag{5.2}$$

with $F_0 > 0$, $0 < \lambda_1(k) \leq 1$; $0 \leq \lambda_2(k) < 2$.

Note that $\lambda_1(k) \equiv 1$, $\lambda_2(k) \equiv 0$ corresponds to the "classical" constant gain algorithm, and $\lambda_1(k) \equiv 1$, $\lambda_2(k) > 0$ corresponds to a "time-decreasing" adaptation gain algorithm. For $\lambda_1(k) = 1$, $\lambda_2(k) = 1$, one have a "least square" type algorithm. Note also that depending on the choice of $\lambda_1(k)$ and $\lambda_2(k)$, one can obtain various laws for the variation of the gain matrix F_k [8], [16].

ν_k in Eq. (5.1) as it will be shown next depends in fact of $\hat{p}(k)$ and therefore, Eq. (5.1) cannot be directly used to update $\hat{p}(k)$. However by an appropriate design of the adaptive system ν_k can be expressed in terms of ν_k^o which is a linear combination of the last "a priori" generalized error ϵ_k^o (called also "a priori" plant-model error or "prediction" error) and the previous values of ϵ_k ($\epsilon_{k-1} \dots \epsilon_{k-n}$). For the remainder of this paper, the designs considered will be such that the relation between ν_k and ν_k^o be of the form:

$$\nu_k = \frac{\nu_k^o}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} . \tag{5.3}$$

For other possible design leading to slightly different expressions for ν_k, see [8].

In the context of S-STURE, an analysis can be made only for the "decreasing" gain case, because the noise which will enter in Eq. (5.1), through ν_k (only for a decreasing gain F_{k-1}, $\hat{p}(k)$ will converge to a fixed value).

The analysis of the M.R.A.C. and S-STURE using a parametric adaptation algorithm of the form (5.1) (with the restriction indicated above) can be done using the EFR method (equivalent feedback representation) and the ODE method (ordinary differential equation) respectively. The basic ideas behind the EFR method is to associate with the equation of the generalized error an equivalent feedback system representation which can be partitioned in a feed-forward linear time invariant block and a time-varying nonlinear feedback block and then the analysis of the stability of this equivalent feedback system is carried on using positivity and hyperstability concepts. For details, see [9], [13], [16].

The basic idea of the ODE method developed by Ljung [12], [17], is to associate with the algorithm (5.1) an ordinary differential equation which will describe the asymptotic properties of the algorithm. Note also that these tools will allow also to give answers to another two problems of importance namely: what will be the behaviour of MRAC in the presence of stochastic disturbances and what will be the behaviour of S-STURE in a deterministic environment. Let's state next two basic results derived from these two methods and which are useful for the analysis of MRAC and S-STURE, using the parametric adaptation algorithm (5.1).

<u>THEOREM 5.1</u> (E.F.R.)

Assume that the parametric adaptation algorithm is given by Eq. (5.1) and (5.2). Assume that the following relation exists between ν_k and ϕ_{k-1}.

$$\nu_k = H(q^{-1})[p - \hat{p}(k)]^T \phi_{k-1} \tag{5.4}$$

where $H(z^{-1})$ is a rational transfer function normalized under the form:

$$H(z^{-1}) = \frac{1 + h_1' z^{-1} + \dots h_\alpha' z^{-\alpha}}{1 + h_1 z^{-1} + \dots h_\beta z^{-\beta}} \; . \tag{5.5}$$

Then:

$$\lim_{k \to \infty} \nu_k = 0 \quad \forall\, \hat{p}(0) - p,\ \nu_0 \tag{5.6}$$

if the transfer function:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2} \tag{5.7}$$

is strictly positive real, where:

$$\max_{0 \leq k < \infty} \lambda_2(k) \leq \lambda < 2\ . \tag{5.8}$$

For the proof of this theorem, see [8], [13].

<u>THEOREM 5.2</u> (O.D.E.) [12], [17]

Assume that the parametric adaptation algorithm is given by Eqs. (5.1) and (5.2) with $\lambda_1(k) = 1$ and $\lambda_2(k) = \lambda_2$.

Suppose that the stationary processes $\{\overline{\phi}_k(\hat{p})\}$, $\{\overline{\nu}_k(\hat{p})\}$ can be defined for all possible values of $\hat{p}_k$.

Assume that:

1) $\overline{\nu}_k(\hat{p}) = H(q^{-1})\overline{\phi}^T_{k-1}(\hat{p})[p^* - \hat{p}] + v_k$ (5.9)

2a) v_k is a sequence of independent normal random variables $(0, \sigma)$

or

2b) $\{\overline{\phi}_{k-1}(\hat{p})\}$ and $\{v_k\}$ are independent stationary stochastic processes.

3) $E\{\overline{\phi}_{k-1}(\hat{p}), \tilde{\phi}_{k-1}(\hat{p})\}[p^* - \hat{p}] = 0$ (5.10)

has a unique solution $\hat{p} = p^*_j(\tilde{\phi}_{k-1}(\hat{p}) = H(q^{-1})\overline{\phi}_{k-1}(\hat{p})$.

Then:

$$\text{Prob}\ \{ \lim_{k \to \infty} \hat{p}(k) = p^* \} = 1 \tag{5.11}$$

if the transfer function:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2} \tag{5.12}$$

is strictly positive real.

For the proof of this theorem, see [13]. Note the similarity between the two theorems. Equation (5.9) becomes identical to Eq. (5.4) when $\hat{p}(k) = \hat{p}$ and $v_k = 0$ and the convergence conditions are exactly the same if the same adaptation algorithm is used.

Note also that Theorem 5.2 can be used for the analysis of M.R.A.C. in the presence of stochastic disturbances. If a M.R.A.C. under consideration verifies in a deterministic environment Theorem 5.1, then v_k in Eq. (5.9) is the image of the disturbance acting upon the M.R.A.C. in the error equation for $\hat{p}_k = \hat{p}$.

Then w.p.1 convergence of $\hat{p}(k)$ to p^* will be achieved if conditions 2 and 3 of Theorem 5.2 are verified.

6. TWO BASIC SCHEMES

We will consider next an MV-STURE as an example of S-STURE and an example of implicit MRAC used for regulation. These schemes will be used then to emphasize the interrelations existing between Implicit and Explicit MRAC and between S-STURE and M.R.A.C.

6.1 The minimum variance-STURE

The equations describing the MV-STURE are briefly reviewed. For details, see [10]. The plant is given by Eq. (2.1) repeated here for convenience:

$$y_k = P_0^T \phi_{k-1} + b_0 u_{k-1} + C(q^{-1}) v_k \tag{6.1}$$

where:

$$P_0^T = [a_1 \ldots a_n,\ b_1 \ldots b_m] \tag{6.2}$$

$$\phi_{k-1}^T = [y_{k-1} \ldots y_{k-n},\ u_{k-2} \ldots u_{k-m-1}] \tag{6.3}$$

$$C(q^{-1}) = [1 - c_1 q^{-1} \ldots - c_n q^{-n}] \quad . \tag{6.4}$$

As mentioned in the introduction, b_o is supposed known and the parameter vector P_0^T as well as $C(q^{-1})$ are *supposed uknown and constant* (a similar development can be done for the case b_o unknown and a basic delay superior to 1). As for the linear case with known parameters the polynomials $B(q^{-1})$ and $C(q^{-1})$ given in Eqs. (2.3) and (6.4) are supposed to have all their zeros inside the unit circle.

The MV-STURE algorithm is conceptually obtained in two steps using an extension of the separation principle.

Step 1: The adaptive predictor

$$\hat{y}_{k/k-1} = \hat{p}_{MV}(k-1)\phi_{k-1} + b_o u_{k-1} \tag{6.5}$$

where:

$$\hat{p}_{MV}^T(k) = [\hat{p}_1(k) \ldots \hat{p}_n(k), \hat{b}_1(k) \ldots \hat{b}_m(k)]$$

$$= [\hat{a}_1(k)-c_1, \ldots \hat{a}_n(k)-c_n, \hat{b}_1(k) \ldots \hat{b}_m(k)] . \tag{6.6}$$

The adjustable parameter vector $\hat{p}_{MV}(k)$ is updated using the algorithm:

$$\hat{p}_{MV}(k) = \hat{p}_{MV}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} \epsilon_k^o \tag{6.7}$$

where:

$$\epsilon_k^o = y_k - \hat{y}_{k/k-1} \tag{6.8}$$

and F_{k-1} is given by (5.2) with $\lambda_1(k) = 1$ and $\lambda_2(k) = \lambda_2$, $0 < \lambda_2 < 2$.

Step 2: Determine a control for the adaptive predictor (6.5) (which will be also applied to the plant) such that the deterministic objective $\hat{y}_{k/k-1} \equiv 0$ be achieved (*as for minimum* variance control with known parameters). From Eq. (6.5), one obtains:

$$u_{k-1} = -\frac{1}{b_o}[\hat{p}_{MV}^T(k-1)\phi_{k-1}] \; . \tag{6.9}$$

But, using (6.9), one has:

$$e_k^o\Big|_{u_{k-1} = \text{Eq. (6.9)}} = y_k \tag{6.10}$$

and the adaptation algorithm (6.7) becomes:

$$\hat{p}_{MV}(k) = \hat{p}_{MV}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} y_k \; . \tag{6.11}$$

Applying Theorem 5.2, one gets the following result (see [12]).

THEOREM 6.1 (Ljung)

The MV-STURE defined by Eqs. (6.1) through (6.4) converges w.p.1 to the minimum variance control, i.e.:

$$\text{Prob}\left\{\lim_{k\to\infty} \hat{p}_{MV}(k) = p_{MV}\right\} = 1 \tag{6.12}$$

where:

$$p_{MV}^T = [a_1 - c_1 \; \ldots \; a_n - c_n, \; b_1 \ldots b_m] \tag{6.13}$$

if the discrete transfer function:

$$H'(z^{-1}) = \frac{1}{C(z^{-1})} - \frac{\lambda}{2} \tag{6.14}$$

is strictly positive real and the estimated parameter vector $\hat{p}_{MV}(k)$ belongs infinitely often to D_S defined:

$$D_S = \{A(z^{-1}) \cdot \hat{B}(z^{-1}) - \hat{A}(z^{-1})B(z^{-1}) = 0 \Rightarrow |z| < 1\} \tag{6.15}$$

where

$$\hat{A}(z^{-1}) = 1 - \hat{p}_1 z^{-1} \ldots - \hat{p}_n z^{-n} \tag{6.16}$$

$$\hat{B}(z^{-1}) = b_o + \hat{b}_1 z^{-1} \ldots + \hat{b}_m z^{-m} \; . \tag{6.17}$$

6.2 An Implicit M.R.A.C.

As mentioned in Section 4, the design of an implicit M.R.A.C. is made in two steps: 1) Design of an adaptive predictor; 2) Design of a control for the predictor (which is also applied to the plant) in accordance with the regulation or tracking objectives, such that the predictor output behaves like the output of a reference model specifying the design objectives.

The process to be controlled is described by:

$$y_k = P_0^T\phi_{k-1} + b_o u_{k-1} \;;\quad y_o \neq 0 \tag{6.18}$$

(the parameter P_o is supposed unknown but constant.)

Step 1: The adaptive predictor ("series-parallel type" is described by:

$$\hat{y}_{k/k-1} = \hat{y}_k^o = \hat{p}^T(k-1)\phi_{k-1} + b_o u_{k-1} + c^T e_{k-1} \tag{6.19}$$

$$\hat{y}_{k/k} = \hat{y}_k = \hat{p}^T(k)\phi_{k-1} + b_o u_{k-1} + c^T e_{k-1} \tag{6.20}$$

where:

$$e_k^o = y_k - \hat{y}_k^o \tag{6.21}$$

$$e_k = y_k - \hat{y}_k \tag{6.22}$$

$$e_{k-1}^T = [e_{k-1} \dots e_{k-n}] \tag{6.23}$$

$$c^T = [-c_1 \dots -c_n] \tag{6.24}$$

($\hat{y}_k^o$ and $\hat{y}_k$ are also called "a priori" and "a posteriori" output respectively [8], [9]).

From Eqs. (6.18), (6.20) and (6.22), one obtains:

$$e_k = [p_o - \hat{p}(k)]^T\phi_{k-1} - c^T e_{k-1} = \frac{1}{C(z^{-1})}[p-\hat{p}(k)]^T\phi_{k-1}. \tag{6.25}$$

Equation (6.25) is of the form of Eq. (5.4) of Theorem 5.1.

Then, if the adaptation algorithm is given by:

$$\hat{p}(k) = \hat{p}(k-1) + F_{k-1}\phi_{k-1}\epsilon_k \tag{6.26}$$

where F_{k-1} is given by (5.2). (Since p_o is supposed unknown and constant, a time-decreasing adaptation gain can be used, i.e. one can choose as for MV-STURE $\lambda_1(k) = 1$ and $\lambda_2(k) = \lambda_2$; $0 < \lambda_2 < 2$) and if:

$$H'(z^{-1}) = \frac{1}{C(z^{-1})} - \frac{\lambda}{2} \tag{6.27}$$

is strictly positive real, one concludes that $\lim_{k \to \infty} \epsilon_k = 0$.

Note also that ϵ_k can be expressed in terms of ϵ_k^o, which is the "a priori" error or a prediction error, when using the algorithm (6.26):

$$\epsilon_k = \frac{\epsilon_k^o}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} \tag{6.28}$$

which introduced in (6.26) gives:

$$\hat{p}(k) = \hat{p}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} \epsilon_k^o . \tag{6.29}$$

<u>Step 2</u>: Computation of the control

The convergence of $\hat{y}_k$ to y_k being assured for all u_k bounded, one chooses u_k such that the following objective be achieved:

$$\hat{y}_{k/k-1} = \hat{y}_k^o \equiv 0 \tag{6.30}$$

(i.e. the desired value of the process output is 0 and this is specified by the output of the predictor which plays the role of the output of an implicit reference model).

Using (6.19), one obtains:

$$u_{k-1} = -\frac{1}{b_o}[\hat{p}^T(k-1)\phi_{k-1} + c^T e_{k-1}] \tag{6.31}$$

but, in this case, $\hat{y}_{k/k-1} \equiv 0$ and therefore:

$$e_k^o = y_k \tag{6.32}$$

$$e_k = y_k - [\hat{p}(k) - \hat{p}(k-1)]^T \phi_{k-1} . \tag{6.33}$$

The adaptation algorithm (6.29) and the control law (6.31) becomes in this case:

$$\hat{p}(k) = \hat{p}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} y_k \tag{6.34}$$

$$u_{k-1} = -\frac{1}{b_o}\{\hat{p}^T(k-1)\phi_{k-1} - \sum_{i=1}^{n} c_i y_{k-i}$$

$$+ \sum_{i=1}^{n} c_i[\hat{p}(k-i) - \hat{p}(k-i-1)]^T \phi_{k-1-i}\}. \tag{6.35}$$

Assuming $|\phi_{k-1}^T F_{k-1}\phi_{k-1}| < M$ and since $\lim_{k\to\infty} e_k = 0$, one has:

$$\lim_{k\to\infty} [\hat{p}(k) - \hat{p}(k-1)]^T \phi_{k-1} = \phi_{k-1}^T F_{k-1}\phi_{k-1} e_k = 0 \tag{6.36}$$

and therefore:

$$\lim_{k\to\infty} u_{k-1} = -\frac{1}{b_o}[\hat{p}^T(k-1)\phi_{k-1} - \sum_{i=1}^{n} c_i y_{k-i}]$$

$$= -\frac{1}{b_o}[\hat{p}_{MV}^T(k-1)\phi_{k-1}] \tag{6.37}$$

where $\hat{p}_{MV}(k-1)$ is given by Eq. (6.6).

Observe that for $\hat{p}(k) = p_o$, the closed loop behaviour will be defined by:

$$C(q^{-1})y_k = 0 \tag{6.38}$$

i.e. the desired poles of the closed loop systems are those of the prediction error equation for $\hat{p}(k) = p_o$. Therefore this scheme achieves the regulation objective specified by Eq. (6.38) and the vector c^T in Eq. (6.19) is chosen in accordance with this objective.

7. EQUIVALENT IMPLICIT AND EXPLICIT M.R.A.C.

The possibility of obtaining equivalent implicit and explicit MRAC has been mentioned in Section 4. Here, we will state precisely the equivalence between the two schemes and we will give the design of an Explicit MRAC which is equivalent to the implicit MRAC presented in Section 6.2.

DEFINITION 7.1: (Equivalence between Explicit and Implicit MRAC): An Explicit MRAC and an Implicit MRAC are equivalent if and only if:

1) the equations for the generalized error ϵ_k are identical,
2) the parametric adaptation algorithms are identical, and
3) the positivity conditions for global asymptotic stability are identical.

For designing the explicit MRAC, which is equivalent to the implicit MRAC given in Section 6.2, one should consider a reference model which have the same output as the adaptive predictor used in the scheme, given in Section 6.2 and one should obtain the same equation for the generalized error.

One defines the "a priori" and "a posteriori" explicit reference model by:

$$x_k^o = 0 \tag{7.1}$$

$$x_k = [\hat{p}(k) - \hat{p}(k-1)]^T \phi_{k-1} \tag{7.2}$$

(for details, see [8], [9]).

One considers the adaptive control law:

$$u_{k-1} = -\frac{1}{b_o}[\hat{p}(k-1)^T \phi_{k-1} + c^T e_{k-1}] \tag{7.3}$$

for the plant given by (6.18) where:

$$e_k^o = y_k - x_k^o \tag{7.4}$$

$$\epsilon_k = y_k - x_k \tag{7.5}$$

$$\epsilon_{k-1}^T = [\epsilon_{k-1} \ldots \epsilon_{k-n}] \tag{7.6}$$

$$c^T = [-c_1 \ldots -c_n]\ . \tag{7.7}$$

From Eqs. (6.18), (7.2), (7.3) and (7.5), one obtains:

$$\epsilon_k = \frac{1}{C(q^{-1})} [p_o - \hat{p}(k)]^T \phi_{k-1} \tag{7.8}$$

which is identical to Eq. (6.26), therefore the first requirement of Definition 6.1 is verified. But, Eq. (7.8) is of the form of Eq. (5.4) from Theorem 5.2. Therefore, using the algorithm (6.26) (or (6.29)), $\lim_{k \to \infty} \epsilon_k = 0$ if the transfer *function of* Eq. (6.27) is strictly positive real. This verifies the third requirement of Def. 7.2. Taking in account that $\epsilon_k^o = y_k$, the parametric adaptation algorithm of Eq. (6.29) becomes identical to that of Eq. (6.34) and one concludes that the Implicit and Explicit MRAC considered are equivalent.

8. <u>ASYMPTOTIC DUALITY BETWEEN S-STURE AND MRAC</u>

<u>DEFINITION 8.1</u>: (Asymptotic Duality between MRAC and S-STURE): An Implicit or Explicit MRAC designed for a deterministic environment is <u>asymptotically dual</u> with respect to a stochastic STURE designed for a stochastic environment if and only if:

1) The adjustable parameter vectors are updated by identical adaptation algorithms (same structure, same observation vector (ϕ) and same generalized error (ϵ_k^o)).

2) The positivity conditions for the global asymptotic stability of the MRAC and for the w.p.1 convergence of the S-STURE are the same.

3) The control laws are asymptotically identical as $k \to \infty$.

Remarks

1) If the control laws are identical for any k, they are called dual.

2) For tracking the point 3 of Definition 8.1 should take in account as in the linear case with known parameters (see Section 3) the change of the reference input in the deterministic case by a white sequence in the stochastic case.

Consider now the MV-STURE given in Section 6.1 and the Implicit MRAC given in Section 6.2. One observes that they verify the conditions of Definition 8.1 (Eq. (6.7) and Eq. (6.29) are identical, Eq. (6.14) and Eq. (6.27) are identical and Eq. (6.9) and Eq. (6.37) are identical).

Same conclusions hold if one compares the MV-STURE given in Section 6.1 and the Explicit MRAC given in Section 7 (as a consequence of the equivalence between the Implicit and Explicit MRAC considered). These results are summarized as follows:

THEOREM 8.1: The Implicit MRAC given in Section 6.2 and the Explicit MRAC given in Section 7 are equivalent and both are asymptotically dual in the sense of Definition 8.1 with respect to the MV-STURE given in Section 6.1.

In fact, given the MV-STURE configuration of Section 6.1, we have constructed the asymptotically dual Implicit and Explicit MRAC configuration. These configurations are new with respect to the various known MRAC configurations. Their originality came from the fact that the generalized error is equal to the process output while in all the other configurations this is never the case (despite that they can have the same objective as it will be shown in Section 10).

9. STOCHASTIC STURE WITH EXPLICIT PREDICTION REFERENCE MODELS

We have seen in Section 3 the connections between linear model following control and linear stochastic (ARMA) model following control and in particular the fact that in both cases, the control objectives can be specified by a reference model and a condition upon the plant-model error. We have seen also in Section 4 the structural similarities between S-STURE and Implicit MRAC. In Sections 7 and 8, the equivalence of an Implicit and an Explicit MRAC which are both asymptotically dual with respect to MV-STURE (which is a particular S-STURE) have been shown.

The natural question which comes up is: Does there exist an equivalent realization of S-STURE which features structural similarities with explicit MRAC? The answer is yes and always the diagram given in Fig. 9.1 can be filled up if the S-STURE under consideration is of the form where the controller parameters are linear explicit function of the estimated parameters of the adaptive predictor. This equivalent realization of S-STURE uses an "explicit prediction reference model" and will be illustrated for the case of the MV-STURE considered in Section 6.1. However, in this particular case, the MV-STURE with "explicit prediction (reference) model" will be indistinguishable from the MV-STURE of Section 6.1 which will be called with implicit prediction (reference) model, because the output of the prediction model is always zero. The scheme is given in Fig. 9.2 and the corresponding equations are:

The explicit prediction model:

$$x_{k/k-1} = 0 \; . \tag{9.1}$$

The plant is given by Eq. (6.1) and the control is given by:

$$u_{k-1} = -\frac{1}{b_o}\left[p_{MV}^T(k-1)\phi_{k-1}\right] \; . \tag{9.2}$$

The prediction error is defined as:

$$e_k^o = y_k - x_{k/k-1} \tag{9.3}$$

and the adaptation algorithm is given by:

$$\hat{p}_{MV}(k) = \hat{p}_{MV}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} e_k^o$$

$$= p_{MV}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} y_k . \tag{9.4}$$

Similar to the deterministic case, one introduces the following definition for equivalence in the stochastic case:

DEFINITION 9.1: (Equivalence between S-STURE with implicit and explicit prediction models): A S-STURE with I.P.M. and a S-STURE with E.P.M. are equivalent if:

1) The equations for the prediction error e_k^o are identical.
2) The parametric adaptation algorithms are identical.
3) The possible convergence points are identical.
4) The positivity conditions for w.p.1 convergence (if they exist) are identical.

It is obvious that the S-STURE with implicit prediction model given in Section 6.1 and the S-STURE with E.P.M. given above are equivalent in the sense of Definition 9.1.

10. THE POSITIVITY PROBLEM AND THE CONVERSE DUAL PROBLEM

The analysis of the MRAC scheme given in Section 6.2 has shown that a condition for global asymptotic stability is that:

$$\frac{1}{C(z^{-1})} - \frac{\lambda}{2} \tag{10.1}$$

be strictly positive real, where $C(z^{-1})$ defines the desired poles of the closed loop. This condition is restrictive (as for the MV-STURE) since it limits drastically the region of the possible poles in the z-domain.

In the MRAC designs, three solutions have been considered in order to overcome this problem:

1) Introduction of a linear compensator acting on the generalized error ϵ_k [8], [9].
2) Modification of the MRAC configuration (i.e. of the reference model) [8].
3) Introduction of appropriate filters for generating the observation vector ϕ_{k-1} [8], [18].

The *first and third solutions are to a certain extent* similar since the objective is to introduce a numerator in the transfer function $H(z^{-1})$ appearing in Eq. (5.4) (respectively $\frac{1}{C(z^{-1})}$ in (10.1)) and we will consider next only the first two solutions.

The first solution applied to the implicit MRAC or explicit MRAC considered in Sections 6.2 and 7 consists of the introduction of a linear compensator $D(z^{-1})$ whose input is ϵ_k and whose output is ν_k. One defines the a priori and a posteriori processed generalized error as:

$$\nu_k^o = \epsilon_k^o + d^T e_{k-1} \tag{10.2}$$

$$\nu_k = \epsilon_k + d^T e_{k-1} = D(q^{-1})\epsilon_k \, . \tag{10.3}$$

The relation between ν_k and ϕ_{k-1} becomes:

$$\nu_k = \frac{D(q^{-1})}{C(q^{-1})} [p_o - \hat{p}(k)]^T \phi_{k-1} \tag{10.4}$$

and using the adaptation algorithm:

$$\hat{p}(k) = \hat{p}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi^T F_{k-1}\phi_{k-1}} \nu_k^o = \hat{p}(k-1) + F_{k-1}\phi_{k-1}\nu_k \,. \tag{10.5}$$

The positivity condition of Theorem 5.1 is satisfied if:

$$\frac{D(z^{-1})}{C(z^{-1})} - \frac{\lambda}{2} \tag{10.6}$$

is strictly positive real. Given a $C(z^{-1})$ which is asymptotically stable, one can always determine an appropriate $D(z^{-1})$ in order to satisfy the positivity condition through the use of the positive real lemma [9].

Consider now an explicit MRAC where the regulation objectives $C(q^{-1})y_k = 0$ are specified by an "explicit" series parallel reference model with a priori and a posteriori output:

$$x_k^o = \sum_{i=1}^{n} c_i y_{k-i} \tag{10.7}$$

$$x_k = x_k^o + [\hat{p}(k) - \hat{p}(k-1)]^T \phi_{k-1} \,. \tag{10.8}$$

Note that the a priori output of the reference model gives the desired prediction value of the process output based on previous output measurements.

The plant is given by:

$$y_k = p_0^T \phi_{k-1} + b_o u_{k-1} \;; \quad y(0) \neq 0 \tag{10.9}$$

and the control is given by:

$$u_{k-1} = -\frac{1}{b_o} [\hat{p}^T(k-1)\phi_{k-1}] \tag{10.10}$$

where:

$$\hat{p}(k) = \hat{p}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} \epsilon_k^o \tag{10.11}$$

$$\epsilon_k^o = y_k - y_k^o . \tag{10.12}$$

The equation for the "a posterior" generalized error is:

$$\epsilon_k = [p_o - \hat{p}(k)]^T \phi_{k-1} - \sum_{i=1}^{n} c_i y_{k-i} = [p_{MV} - \hat{p}(k)]^T \phi_{k-1} \tag{10.13}$$

where p_{MV} is given by (6.13).

Applying Theorem 5.1, one finds that using the adaptation algorithm (10.11), $\lim_{k \to \infty} \epsilon_k = 0$ without any positivity condition to be satisfied because in this case:

$$H'(z^{-1}) = 1 - \frac{\lambda}{2} > 0 . \tag{10.14}$$

One can also immediately construct an implicit MRAC equivalent to the above explicit one and then one can ask what are the S-STURE with IPM or EPM which is dual to the MRAC considered?

The result of this investigation leads to the following S-STURE with IPM and E.P.M.

<u>S-STURE with I.P.M.</u>:

The process and its environment is given by:

$$y_k = p_0^T \phi_{k-1} + b_o u_{k-1} + v_k \tag{10.15}$$

where v_k is a sequence of independent normal random variable $(0, \sigma)$. The objective to be asymptotically achieved is: $C(q^{-1})y_k = v_k$.

The adaptive predictor is given by:

$$\hat{y}_{k/k-1} = \hat{p}_0^T(k-1)\phi_{k-1} + b_o u_{k-1} . \tag{10.16}$$

The adaptation algorithm is given by:

$$\hat{p}_0(k) = \hat{p}_0(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} \epsilon_k^o . \tag{10.17}$$

The control should be such that $\hat{y}_{k/k-1} = \sum_{i=1}^{n} c_i y_{k-i}$ which leads to:

$$u_{k-1} = -\frac{1}{b_o}\left[p_0^T(k-1) - \sum_{i=1}^{n} c_i y_{k-i}\right] . \qquad (10.18)$$

<u>S-STURE with E.P.M.</u>:

The explicit prediction model will be given by:

$$x_{k/k-1} = \sum_{i=1}^{n} c_i y_{k-i} . \qquad (10.19)$$

The plant is given by (10.15) and the control law by:

$$u_{k-1} = -\frac{1}{b_o}\left[\hat{p}(k-1)\phi_{k-1}\right] . \qquad (10.20)$$

The adaptation algorithm is given by:

$$\hat{p}(k) = \hat{p}(k-1) + \frac{F_k \phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1} \phi_{k-1}} e_k^o . \qquad (10.21)$$

Using Theorem 5.2, the both STURE converge w.p.1 to the linear stochastic controller assuring $C(q^{-1})y_k = v_k$ for a stochastic disturbance v_k.

Note that in this case the MRAC and S-STURE are <u>dual</u> because the control laws are identical for any k. However, the main conclusion of this analysis is that the positive real condition can be removed in the deterministic context for achieving the same control objectives but in the stochastic case, the removing of the positivity condition correponds to a change of the nature of the disturbance and of the control objectives (or only one of two).

A similar conclusion holds when a linear compensator is introduced in the S-STURE given in Section 6.1. Defining $\nu_k = D(q^{-1})\varepsilon_k$, this leads to the condition (10.6) for w.p.1. convergence if v_k is replaced by $w_k = \frac{1}{D(q^{-1})} v_k$ and the control objective becomes $y_k = w_k$ (or $D(q^{-1})y_k = v_k$).

11. THE NOISE EFFECT UPON M.R.A.C AND THE DUALITY MRAC-STURE

One of the important questions in designing MRAC is their behaviour in the presence of stochastic disturbances acting upon the plant. The choice between various possible configurations [8] or the development of new configurations will depend on their desired properties in a stochastic environment.

The analysis of MRAC in a stochastic environment can be done using Theorem 5.2. Little work has been done in this area [1] but work is in progress. To illustrate this aspect, one considers the implicit MRAC given in Section 6.2 (or which is equivalent to the Explicit MRAC given in Section 7).

The plant in this case will be described by:

$$y_k = p_0^T \phi_{k-1} + b_o u_{k-i} + C'(q^{-1}) v_k \tag{11.1}$$

where $C'(q^{-1})$ is given by:

$$C'(q^{-1}) = 1 - \sum_{i=1}^{n} c_i' q^{-i} \tag{11.2}$$

and v_k is a sequence of independent normal random variables $(0, \sigma)$.

First remark is that if $c_i' = c_i$, the deterministic control assures in the mean time the minimum variance control and a straightforward analysis show that because of the duality this scheme behaves exactly as the MV-STURE of Section 6.1 (*the transient control signals depending on* $(\hat{p}(k) - \hat{p}(k-1))$ disappears in the analysis using Theorem 5.2 since $\hat{p}(k) = \hat{p}(k-i) = \hat{p}$). Therefore, we will consider next the case $c_i \neq c_i'$ and we will sketch briefly the analysis using Theorem 5.2.

With the new Eq. (11.1), for the plant and its environment, one gets:

$$\epsilon_k = [p_o - \hat{p}(k)]^T \phi_{k-1} + \sum_{i=1}^{n} c_i \epsilon_{k-i} - \sum_{i=1}^{n} c_i' v_{k-i} + v_k . \tag{11.3}$$

Defining now the stationary sequences $\{\bar{\epsilon}_k(\hat{p})\}$ and $\{\bar{\phi}_{k-1}(\hat{p})\}$ for $\hat{p}(k) = \hat{p}$, Eq. (11.3) becomes:

$$\bar{\epsilon}_k(\hat{p}) = [p_o - \hat{p}]^T \bar{\phi}_{k-1}(\hat{p}) + \sum_{i=1}^{n} c_i \bar{\epsilon}_{k-1}(\hat{p}) - \sum_{i=1}^{n} c'_i v_{k-1} + v_k \; . \tag{11.4}$$

But, for $\hat{p}(k) = \hat{p}$:

$$e_k^o(\hat{p}) = \bar{\epsilon}_k(\hat{p}) = \bar{y}_k(\hat{p}) \tag{11.5}$$

and Eq. (11.4) becomes:

$$\bar{\epsilon}_k(\hat{p}) = \bar{y}_k(\hat{p}) = [p_{MV}^T - \hat{p}]\bar{\phi}_{k-1}(p) - \sum_{i=1}^{n} c'_i v_{k-i} + v_k \tag{11.6}$$

where p_{MV} is given by (6.6). Adding and subtracting in the right-hand side $\pm \sum_{i=1}^{n} c'_i \bar{y}_{k-i}(\hat{p})$, one obtains, taking into account (11.5):

$$\bar{\epsilon}_k(\hat{p}) = \bar{y}_k(\hat{p}) = [p^* - \hat{p}]\bar{\phi}_{k-1}(\hat{p}) + \sum_{i=1}^{n} c'_i \bar{\epsilon}_k(\hat{p})$$

$$- \sum_{i=1}^{n} c'_i v_{k-i} + v_k \tag{11.7}$$

and finally:

$$\bar{\epsilon}_k(\hat{p}) = \frac{1}{C'(q^{-1})} [p^* - \hat{p}]\bar{\phi}_{k-1}(\hat{p}) + v_k \tag{11.8}$$

where:

$$p^* = [a_1 + c_1 - c'_n, \; \ldots \; a_n + c_n - c'_n, \; b_1 \; \ldots \; b_m] \; . \tag{11.9}$$

Since v_k is white, assuming that also condition 3 of Theorem 5.2 is satisfied (similar analysis as for S-STURE [12]), one concludes that w.p.1 convergence of $\hat{p}(k)$ to p^* will occur if:

$$\frac{1}{C'(z^{-1})} - \frac{\lambda}{2} \tag{11.10}$$

is strictly positive real and one deduces from (6.35) that the control will be as $k \to \infty$:

$$\text{Prob}\{\lim_{k \to \infty} u_{k-1} = -\frac{1}{b_o}[(p^*)^T\phi_{k-1} - \sum_{i=1}^{n} c_i y_{k-i}]$$

$$= -\frac{1}{b_o}[(p'_{MV})^T\phi_{k-1}]\} = 1 \tag{11.11}$$

where:

$$(p'_{MV})^T = [a_1 - c'_1 \; \ldots \; - a_n - c'_n, \; b_1 \; \ldots \; b_m] \;. \tag{11.12}$$

Note that a bias appears with respect to the desired control parameter vector for the deterministic situation which is:

$$p^T_{MV} = [a_1 - c_1, \; \ldots \; a_n - c_n, \; b_1 \; \ldots \; b_m] \tag{11.13}$$

and this bias will depend on the difference $c_i - c'_i$.

On the other hand, the new convergence point corresponds to the convergence point of the dual S-STURE for the same stochastic disturbance.

One has then the following important conclusion:

<u>THEOREM 11.1</u>: A M.R.A.C. in the presence of a stochastic disturbance of the same structure as that considered for its dual S-STURE will behave asymptotically as its dual S-STURE.

The term "asymptotically" comes from the fact that in most of the cases, the control laws will become only asymptotically identical because of the transient adaptation signals which are used in MRAC (but this aspect will be further investigated next).

12. <u>THE S-STURE IN A DETERMINISTIC ENVIRONMENT AND THE DUALITY MRAC-S-STURE</u>

Consider the MV-STURE described in Section 1. Assume $v_k = 0$, $c_i = 0$, $i = 1,\ldots,n$. Then, the MV-STURE operates in a deterministic environment and a deterministic stability analysis should be considered. Note that for $v_k = 0$, $c_i = 0$, $i = 1,\ldots,n$, the plant and the predictor are identical to that of the

implicit MRAC given in Section 6.2 and since the $c_i \equiv 0$, the transient terms in the control law of the implicit MRAC disappear. The transfer function $\frac{1}{C(z^{-1})} = 1$, and one concludes that both schemes become equivalent and will be globally asymptotically stable. However the disadvantage of the S-STURE in such a situation is that all the closed loop poles will be at $z = 0$, and this means a one step response which is in most of the cases undesirable because of the magnitude of the resulting control.

Therefore in a certain way, the desired performance of S-STURE in a deterministic environment should be specified by the designer and this should not modify the behaviour of the S-STURE in a stochastic environment. From the analysis carried on in Section 11, this can be achieved by replacing the S-STURE by its dual M.R.A.C. and of course since the $c_i \neq 0$ (which will specify the desired behaviour in the deterministic environment) the transient adaptation terms in the control law should be added in order to satisfy Theorem 5.1.

13. A COMBINED MRAC-S-STURE SCHEME

We will give next a scheme which illustrates how an adaptive scheme behaving as a MRAC in a deterministic environment and a S-STURE in a stochastic environment can be obtained (it summarizes the previous analysis given in Sections 11 and 12).

The plant in a deterministic environment is described by:

$$y_k = p_0^T \phi_{k-1} + b_o u_{k-1} \; ; \quad y_o \neq 0 \tag{13.1}$$

and in a stochastic environment is described by:

$$y_k = p_0^T \phi_{k-1} + b_o u_{k-1} + C'(q^{-1}) v_k \tag{13.2}$$

where v_k is a sequence of independent normal random variables $(0, \sigma)$ and:

$$C'(q^{-1}) = 1 - c_1'q^{-1} \dots - c_n'q^{-n} . \tag{13.3}$$

The adaptive predictor:

$$\hat{y}_{k/k-1} = \hat{y}_o(k) = \hat{p}^T(k-1)\phi_{k-1} + b_o u_{k-1} + c^T e_{k-1} \tag{13.4}$$

$$\hat{y}_{k/k} = \hat{y}_k = \hat{p}^T(k)\phi_{k-1} + b_o u_{k-1} + c^T e_{k-1} \tag{13.5}$$

where:

$$e_k^o = y_k - \hat{y}_{k/k-1} \tag{13.6}$$

$$e_k = y_k - \hat{y}_{k/k} \tag{13.7}$$

$$e_{k-1}^T = [e_{k-1} \dots e_{k-n}] \tag{13.8}$$

$$c^T = -c_1 \dots -c_n \tag{13.9}$$

where the c_i defines the polynomial

$$C(q^{-1}) = 1 - c_1 q^{-1} \dots - c_n q^{-n} \tag{13.10}$$

which specifies the desired performance in a deterministic environment.

The control law is given by:

$$u_{k-1} = -\frac{1}{b_o}\{\hat{p}(k-1)\phi_{k-1} - \sum_{i=1}^{n} c_i y_{k-i} + \sum_{i=1}^{n} c_i[\hat{p}(k-i) - \hat{p}(k-i-1)]^T \phi_{k-1-i}\} \tag{13.11}$$

and the parametric adaptation algorithm is given by:

$$\hat{p}(k) = \hat{p}(k-1) + \frac{F_{k-1}\phi_{k-1}}{1 + \phi_{k-1}^T F_{k-1}\phi_{k-1}} y_k . \tag{13.12}$$

With respect to the S-STURE, we note the introduction of the term $c^T e_{k-1}$ in the predictor and of the transient term in the control law which come from the deterministic analysis. As simulations have shown the introduction of these additional terms are useful for speeding up the convergence.

The above scheme has the following properties:

1) In a deterministic context, $\lim_{k \to \infty} \epsilon_k = 0$ if

$$\frac{1}{C(z^{-1})} - \frac{\lambda}{2} \qquad (13.14)$$

is strictly positive real and the desired closed loop poles are specified by $C(z^{-1}) = 0$.

2) In a stochastic environment, this scheme converges w.p.1. to the minimum variance control if:

$$\frac{1}{C'(z^{-1})} - \frac{\lambda}{2} \qquad (13.15)$$

is strictly positive real under the assumption that $\hat{p}(k)$ does not leave the domain allowing to define the stationary processes $\{\overline{\phi}_{k-1}(\hat{p})\}$ and $\{\overline{\epsilon}_k(\hat{p})\}$ (see Section 6.1).

Of course, an equivalent explicit MRAC can be defined having the same properties and the same reasonment can be extended to other configurations.

14. CONCLUSIONS

It was shown in this paper that:

1) The duality existing between linear stochastic control and linear deterministic control exists also between the S-STURE (where the objective is to have the output of the process described by a certain ARMA model) and the MRAC (where the objective is to have the output of the process satisfying a certain difference equation).

2) The implicit and explicit MRAC can be equivalent.

3) The S-STURE can be interpreted as stochastic MRAC where the reference model (implicit or explicit) is replaced by prediction models (explicit or implicit).

4) The current used realization of S-STURE are of the type using implicit prediction reference models but equivalent S-STURE with explicit prediction reference models can be defined.

5) The duality properties of S-STURE and MRAC have been exploited for the analysis of MRAC in a stochastic environment and of S-STURE in a deterministic environment.

6) The duality analysis has led to a new configuration of MRAC assuring the same regulation objectives as known MRAC configurations but which have a different behaviour in a stochastic environment.

7) A method for constructing an adaptive scheme which behaves as a given MRAC in a deterministic environment and as a given S-STURE in a stochastic environment has been indicated and illustrated by an example.

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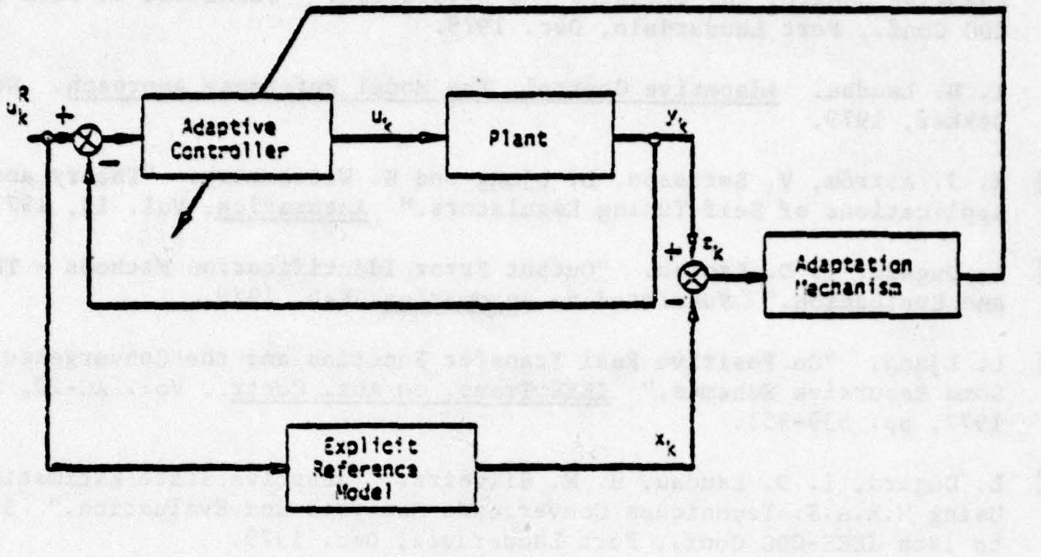


FIGURE 4.1

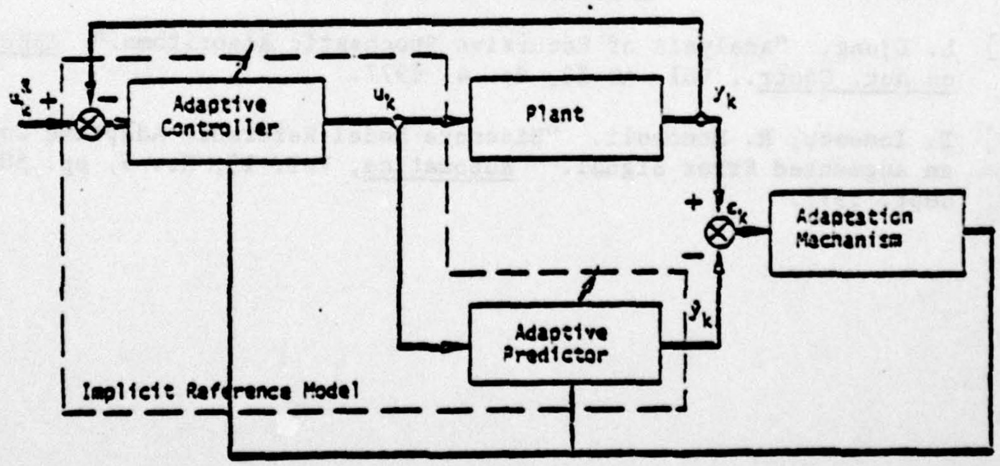


FIGURE 4.2

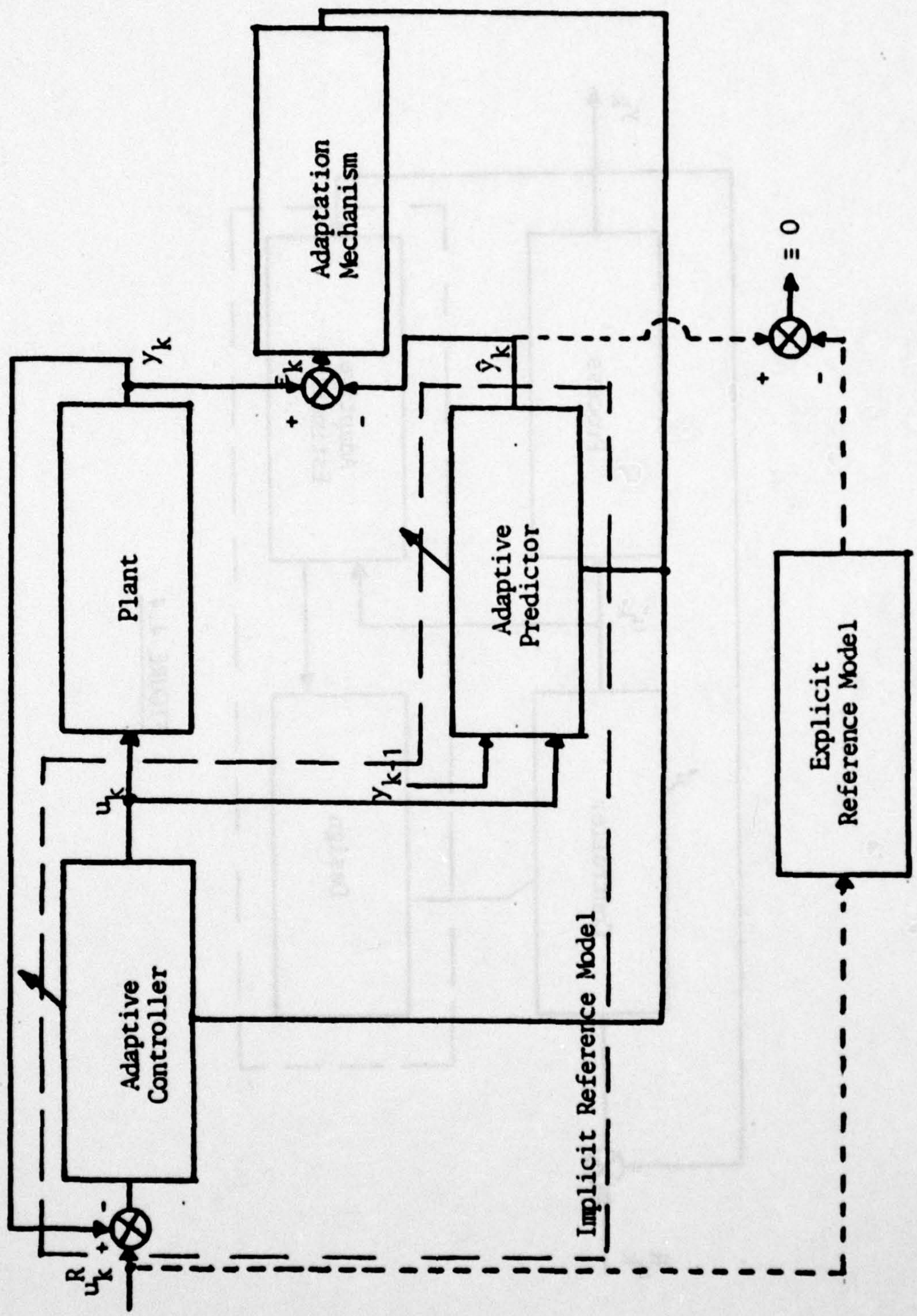

Adaptive Controller
Plant
Adaptation Mechanism
Adaptive Predictor
Explicit Reference Model
Implicit Reference Model
u_k^R
u_k
y_k
$\hat{y}_k$
ε_k
y_{k-1}
$\equiv 0$
+
-

FIGURE 4.3

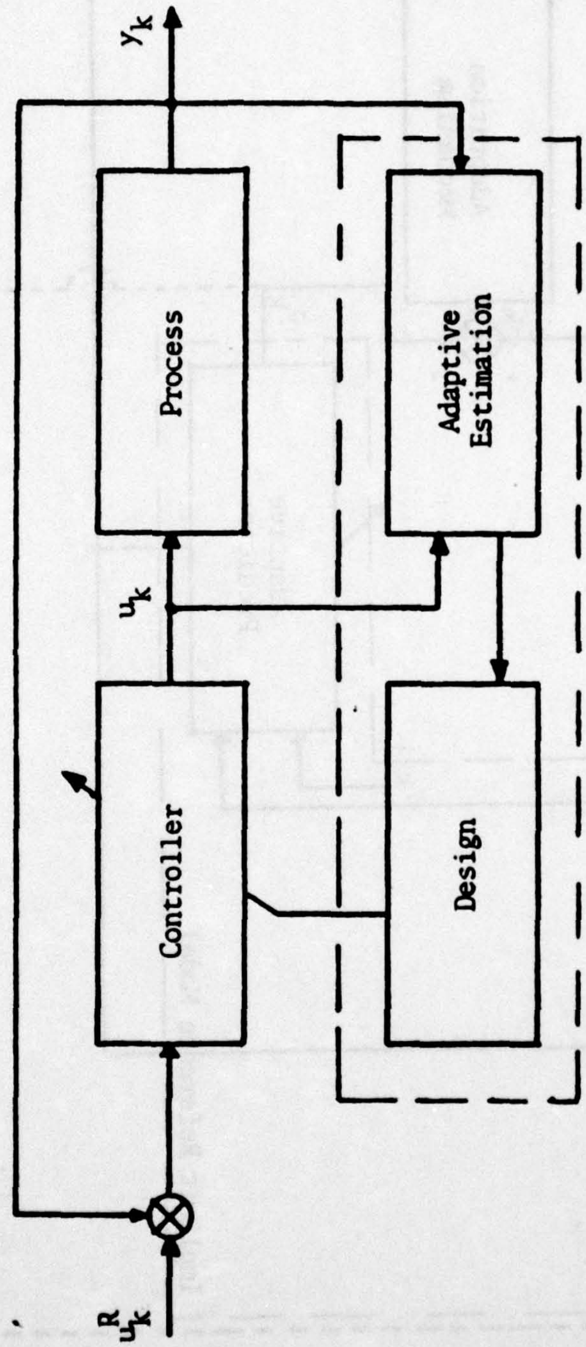


FIGURE 4.4

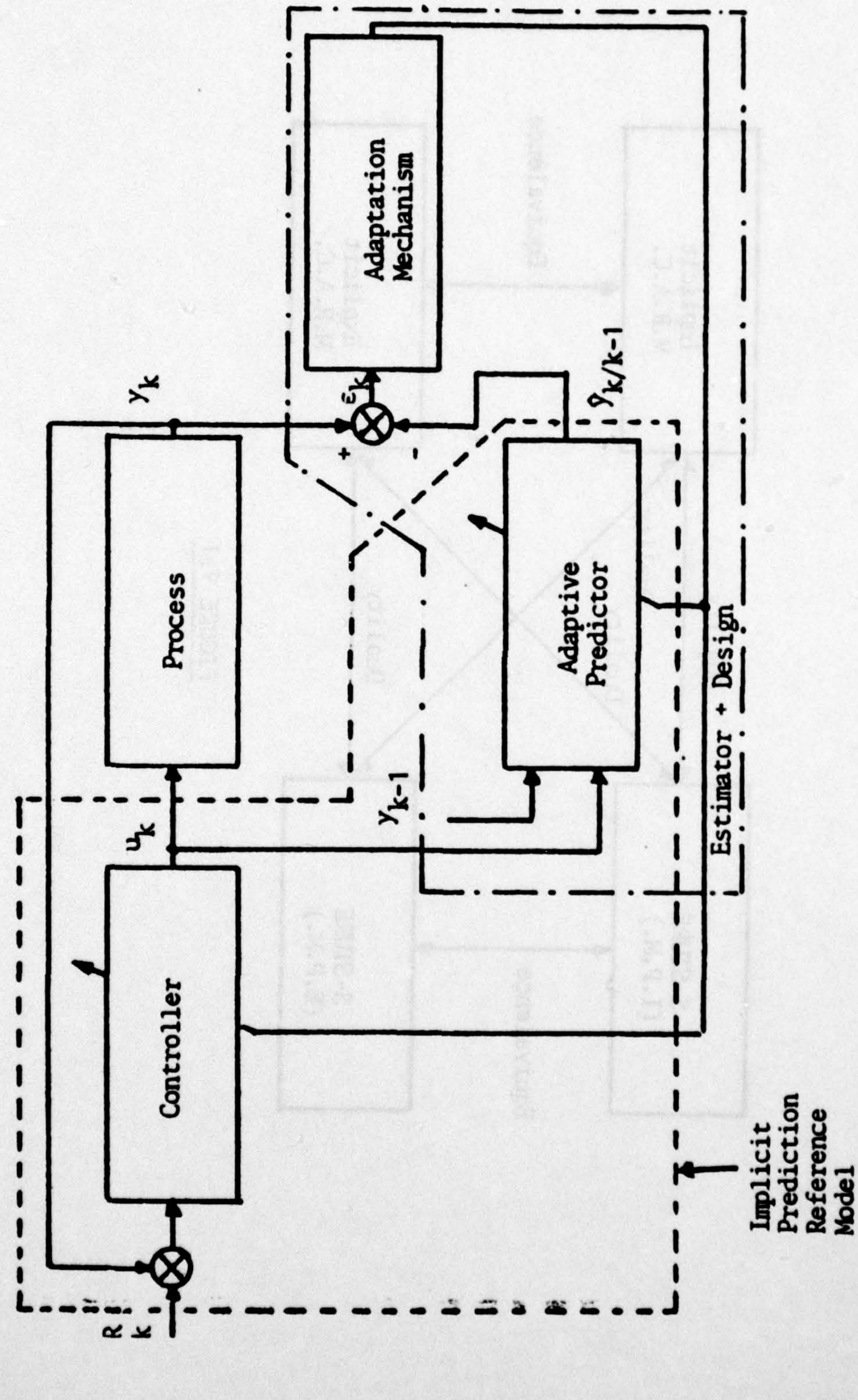

R_k
Controller
u_k
Process
y_k
ε_k
+
-
Adaptation Mechanism
$\hat{y}_{k/k-1}$
Adaptive Predictor
y_{k-1}
Estimator + Design
Implicit Prediction Reference Model

FIGURE 4.5

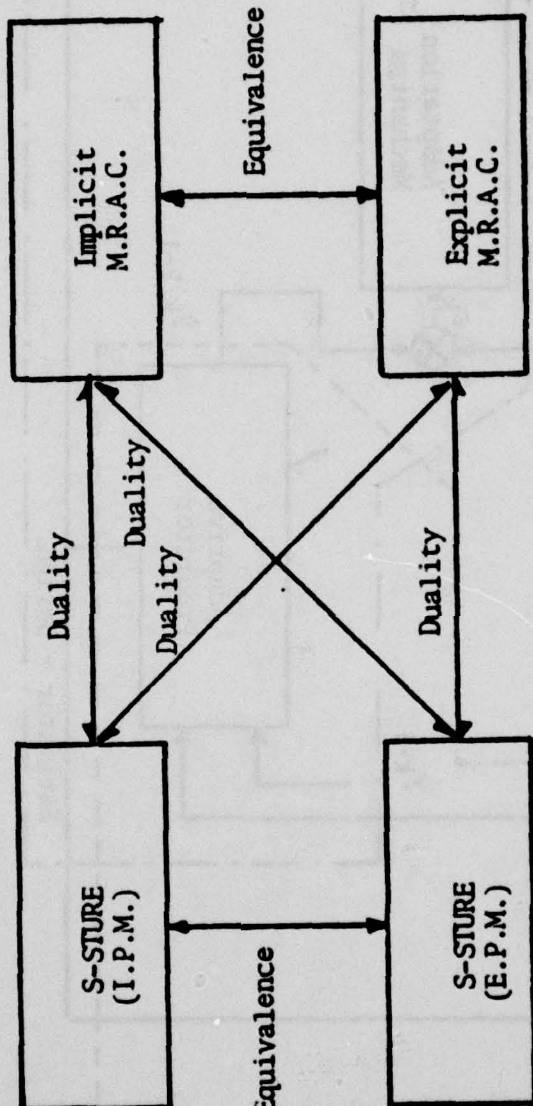
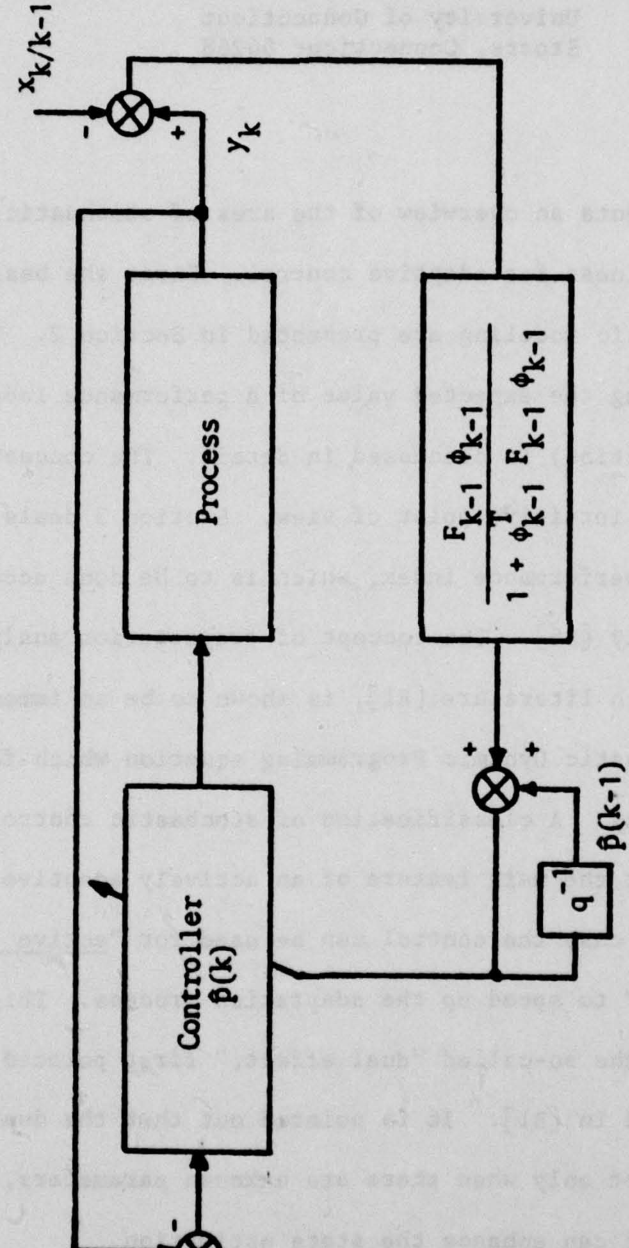

Implicit M.R.A.C.
Explicit M.R.A.C.
S-STURE (I.P.M.)
S-STURE (E.P.M.)
Equivalence
Equivalence
Duality
Duality
Duality
Duality

FIGURE 9.1



$x_{k/k-1}$
y_k
Process
Controller
$\beta(k)$
$\frac{F_{k-1}\,\Phi_{k-1}}{1+\Phi^T_{k-1}\,F_{k-1}\,\Phi_{k-1}}$
q^{-1}
$\beta(k-1)$

FIGURE 9.2

STOCHASTIC ADAPTIVE CONTROL OVERVIEW

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1. Introduction

This paper presents an overview of the area of stochastic control with emphasis on its usefulness for adaptive control. First the basic assumptions related to probabilistic modeling are presented in Section 2. The Bayesian approach of extremizing the expected value of a performance index (in general minimizing a loss function) is discussed in detail. The concept of learning is introduced from an intuitive point of view. Section 3 deals with the extremization of the performance index, which is to be done according to the Principle of Optimality [B6]. The concept of preposterior analysis, known in the Operations Research literature [R1], is shown to be an immediate consequence of the Stochastic Dynamic Programming equation which follows from the Principle of Optimality. A classification of stochastic control laws is presented that points out the main feature of an actively adaptive control algorithm. In such a case the control can be used for "active (control aided) information gathering" to speed up the adaptation process. This is possible when the control has the so-called "dual effect," first pointed out in [F1] and rigorously defined in [B1]. It is pointed out that the dual effect of the control can be used not only when there are unknown parameters, but also in their absence, when it can enhance the state estimation.

Section 4 discusses a number of stochastic adaptive control algorithms: the Heuristic Certainty Equivalence Approach, the Self-Tuning Regulator, the Multiple Model Weighted (partitioned) Adaptive Control and a Closed-Loop Dual

Control. It is shown how a decomposition of the optimum cost obtained in the Dual Control algorithm by a suitable approximation of the Dynamic Programming points out explicitly the Caution and Probing effects caused by the uncertainty in the problem. The application of the dual control method to a missile guidance problem with no unknown parameters but with nonlinear structure illustrates how the control can be successfully used to enhance estimation.

2.0 The Basic Modeling Assumptions in Stochastic Control

In stochastic control, modeling of the uncertainty is done as follows. Imperfect information is summarized in probabilistic form:

(i) random variables - unknown parameters or state

(ii) random processes or sequences in discrete-time disturbances

The system equations are

$$x(k+1) = f_k[x(k), u(k), \theta, v(k)] \tag{2.1}$$

$$y(k) = h_k[x(k), w(k)] \tag{2.2}$$

where

$x(k)$ - state vector at time k

θ - unknown parameters with a prior pdf

$u(k)$ - control (decision variable)

$v(k)$ - state equation disturbance (process noise)

$y(k)$ - measurement

$w(k)$ - measurement noise.

For example, a linear system with unknown parameters is given by

$$x(k+1) = A(\theta)x(k) + B(\theta)u(k) + v(k) . \tag{2.3}$$

In the Bayesian approach the goal is to minimize the expected value of a loss function. In order to be able to obtain the expected value of the loss function every variable this function depends upon must be

(i) deterministic (i.e., known perfectly), or

(ii) random - a pdf has to be attached to all the random variables or processes entering into the description of the system.

There are other approaches (less common) like the minimax [S2] and worst distribution.

2.1 The Bayesian Approach for Discrete Time Stochastic Control

In this approach one considers the dynamic model of system given by

$$x(k+1) = f_k[x(k),\ u(k),\ v(k)] \qquad k = 0,1,\ldots \tag{2.4}$$

where unknown parameters are included in the state vector - they might be time-varying.

The information at the start of the process consists of the joint pdf of the initial state and the sequence of disturbances (process noise).

The cost function is

$$C(0,X^N,U^{N-1}) = C_N[x(N)] + \sum_{k=0}^{N-1} C_k[x(k),\ u(k)] \tag{2.5}$$

where

$$X^N = \{x(k)\}_{k=0}^{N} \quad ; \qquad U^{N-1} = \{u(k)\}_{k=0}^{N-1} . \tag{2.6}$$

Remarks:

1. In some problems the disturbance might also enter into the cost.
2. The terminal time can be
   (i) fixed
   (ii) a random variable (depending on the state)
   (iii) a decision variable.

The expected cost is

$$J = E[C] \tag{2.7}$$

and our problem is

$$\min_{U^{N-1}} J \quad . \tag{2.8}$$

Remark:

The minimization of the expected cost implies that we want to find the optimal policy

(i) over all possible initial conditions.

(ii) over all possible values of the unknown parameters.

(iii) over all possible disturbance sequences.

2.2 The Concept of Learning

If the system to be controlled has some unknown parameters this initial uncertainty can be modelled by a prior pdf $p(\theta|I^0)$.

The initial control $u(0)$ *will account for the fact that it is applied to* a system with parameter θ "drawn" from the prior distribution.

If the parameter θ is time-invariant one can reduce the initial uncertainty about its true value in the course of the control process - the controller can "learn" it.

Thus, as new information is gathered via feedback, the controller can adapt itself to the particular system it is controlling.

There are some fundamental questions related to this concept of learning:

(i) How much is the performance degradation because of the parameter uncertainty?

(ii) Can the uncertainty be reduced during the control process?

(iii) If the uncertainty can be reduced, can the control be used to reduce faster the uncertainty?

3.0 The Principle of Optimality for Stochastic Problems and the Stochastic Dynamic Programming

The basic tool to solve stochastic control problems is the Principle of Optimality: at any time, whatever the present information set and past decisions, the remaining decisions must constitute an optimal policy with regard to the present information set.

In the deterministic case, the state summarizes the past. In the stochastic case, the information set is what the controller knows about the system:

$$I^k = \{Y^k, U^{k-1}\} = \{I^{k-1}, y_k, u_{k-1}\} \quad . \tag{3.1}$$

The problem is to find

$$\min_{U^{N-1}} E[C(0, X^N, U^{N-1})|I^o] \triangleq J^*(0, I^o) \tag{3.2}$$

with

$$u_k = u_k(I^k) \quad . \tag{3.3}$$

From the Principle of Optimality the last decision must be optimal with regard to the information state available when it has to be computed

$$\min_{u_{N-1}} E(C|I^{N-1}) \tag{3.4}$$

where C is the cost for the entire problem.

The next to the last decision u_{N-2}

(i) must be optimal w.r.t. I^{N-2}, and

(ii) is made knowing that u_{N-1} will be optimal w.r.t. I^{N-1}, i.e., it is obtained from

$$\min_{u_{N-2}} E[\min_{u_{N-1}} E(C|I^{N-1})|I^{N-2}] \quad . \tag{3.5}$$

Note that the outside averaging is over y_{N-1}.

Thus the optimal expected cost for the N-step problem is obtained from a sequence of nested expectations and minimizations

$$J^*(0, I^o) = \min_{u_o} E\{\ldots \min_{u_{N-2}} E[\min_{u_{N-2}} E(C|I^{N-1})|I^{N-2}] \ldots |I^o\} . \quad (3.6)$$

The general stochastic dynamic programming equation is, for an additive cost,

$$J^*(k, I^k) = \min_{u_k} E[C_k(x_k, u_k) + J^*(k+1, I^{k+1})|I^k] \quad (3.7)$$

with end condition

$$J^*(N, I^k) = E[C_N(x_N)|I^N] . \quad (3.8)$$

Note that

$$E[J^*(k+1, I^{k+1})|I^k]$$

$$= \int f(y_{k+1}, y_k, \ldots, y_o) p(y_{k+1}|y_k, \ldots, y_o) dy_{k+1} \quad (3.9)$$

i.e., at each iteration one has to average over the next observation. Thus the optimal control depends on

(i) the current information $u_k = u_k(I^k)$.

(ii) the prior statistical description of the future posterior information

$$p(y_{j+1}|I^j) \qquad j \geq k .$$

This is preposterior analysis [R1] which can be paraphrased as "know how to use what you know as well as what you know about what you shall know," and is a consequence of the Principle of Optimality.

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3.1 Types of Algorithms in Stochastic Control

The classification presented below can be made for stochastic control algorithms.

Types of algorithm \ Features	Utilization of real-time observations	Utilization of the statistical description of future observations
Open-loop (OL)		
Feedback (F)	X	
Closed-loop (CL)	X	X

A CL algorithm "knows" that the loop will stay closed throughout the process (F + PPA) (feedback + preposterior analysis).

An OL algorithm is never optimal for a stochastic problem.

The optimum is in general of the CL type with some exceptions when it is of the F type.

The Open-loop-optimal feedback (OLOF) policy

(i) computes the control under the assumption that no future observations will be available (OL) but

(ii) when observations are made they are utilized by the controller to update its information about the system.

This controller belongs to the F class according to the above classification.

The m-measurement-optimal feedback policy computes the control assuming measurements will be available only at the next m sampling times ($m=0 \Rightarrow$ OLOF).

The usefulness of the F/CL distinction is in the following. When the optimum stochastic control is not known for a class of problems one can use suboptimal algorithms of the feedback type or closed-loop type.

To be as close as possible to the optimum, it is important to realize the distinction between F and CL and to be able to obtain an approximation of the stochastic dynamic programming that has the CL property.

3.2 The Control's Dual Effect

If the uncertainty of the state (which includes possibly unknown system parameters) of a stochastic system depends on past control values, the control is said to have a dual effect [F1, B1].

The definition of the dual effect is as follows: The information set at time k is

$$I^k = \{Y^k, U^{k-1}\} . \quad (3.10)$$

The state estimate (conditional mean) is

$$\hat{x}_{k|k} = E\{x_k | I^k\} . \quad (3.11)$$

The covariance of the state at time k is

$$\Sigma_{k|k} = E\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})' | I^k\} . \quad (3.12)$$

Then, if $\Sigma_{k|k}$ does not depend on U^{k-1} the control has no dual effect (of second order) - the control is neutral.

The implications of the dual effect are as follows:

1. Active Information Gathering (Probing)
   - If the control has a dual effect, such that it can reduce some uncertainty, this might be used to improve the overall performance.
   - Only a closed loop control, by anticipating future feedback, can assess the "value of future information" and do a tradeoff between
     (i) control action
     (ii) information gathering to improve the accuracy of subsequent control actions.

2. Caution

   • Due to the inherent uncertainties the controller has to be "cautious" not to increase the effect of the existing uncertainties on the cost.

3.3 Adaptive and Dual Control

The question of what is Adaptive Stochastic Control as discussed at the 1976 CDC [B2] reflected the following points of view:

1. The controller's actions are based on a model of the system that is updated in real time (because initial information is poor or system changes). This has a hierarchical structure of feedback

   (i) lower level: feedback control based upon current model

   (ii) higher level: feedback is used to update the model.

2. One is faced with a nonlinear stochastic control problem which has to be approximated. Adaptive control is a method of approach for the control of systems when the exact formulation is too complex.

A learning system is one which, while operating in a stochastic environment, can reduce the uncertainty in its description as the process evolves [S1]. This is a similar definition to the first one above.

Stochastic adaptive control can be classified as follows:

1. Passively adaptive control where the controller learns about the system but does not anticipate subsequent learning from future feedback. Learning is therefore accidental (passive) - from past "mistakes." Such a controller belongs to the feedback class.

2. Actively adaptive control - the controller learns about the system and anticipates subsequent learning from future feedback. Then learning is enhanced by use of the dual effect - the controller experiments

(probes) to improve the accuracy of information about the model.

Such a controller belongs to the closed-loop class.

3.4 Dual Effect of the Control and General Nonlinear Problems

The dual effect, if accounted for by an adaptive controller, can make it into actively adaptive. The main class of problems where this approach can be used is the control of linear systems with unknown parameters.

In other problems, e.g., linear systems (without parameter uncertainties) and with nonlinear measurement the control has in general a *dual effect*. In such a case one cannot talk about an adaptive control but the control can still enhance the state estimation accuracy [T3, C1].

Such a problem is encountered in homing missile guidance - a dual control can then:

(i) improve performance for given sensor accuracy, or

(ii) lower sensor accuracy requirement for given performance.

4.0 Some Adaptive Stochastic Control Algorithms

4.1 The Heuristic Certainty Equivalence Approach

A linear system with unknown parameters is considered

$$x_{k+1} = F_k(\theta)x_k + G_k(\theta)u_k + v_k \tag{4.1}$$

with linear observations

$$y_k = H_k x_k + w_k \ . \tag{4.2}$$

At time k one has $\hat{x}_{k|k}$ and $\hat{\theta}_k$. A common (and approximate) method of obtaining these estimates is via the Extended Kalman Filter (EKF).

The control algorithm consists of the following. Using the parameter estimate at t_k one has:

$$\hat{F}_{j|k} = F_j(\hat{\theta}_k) \qquad \hat{G}_{j|k} = G_j(\hat{\theta}_k) \qquad j = k,\ldots,N-1 . \tag{4.3}$$

With this one computes the gain L_k from the standard LQ problem as if

$$F_j = \hat{F}_{j|k} \, , \; G_j = \hat{G}_{j|k} \qquad j = k,\ldots,N-1 . \tag{4.4}$$

All the uncertainties in F, G are ignored (HCE). The control

$$u_k = - L_k \hat{x}_{k|k} \tag{4.5}$$

is then applied. At t_{k+1} a new estimate of the parameters $\hat{\theta}_{k+1}$ is obtained and the procedure is repeated.

A recent study [C2] showed stability for an ARMAX system with unknown parameters controlled by such an algorithm when the parameter identification was done via stochastic approximation.

4.2 Self-Tuning Regulator (STURE)

An input-output model with white noise e_k is considered

$$y_k = - A(q^{-1})y_k + B(q^{-1})u_k + C(q^{-1})e_k . \tag{4.6}$$

The cost criterion is minimum variance

$$\min_{u_k} E[y_{k+1}^2 | I^k] . \tag{4.7}$$

In the special case, where C = 1

$$A = \sum_{i=1}^{n} a_i q^{-i} \qquad B = \sum_{i=1}^{n} b_i q^{-i} . \tag{4.8}$$

The optimal minimum variance control (with known parameters) is

$$u_k^{MV} = \frac{A(q^{-1})}{B(q^{-1})} y_k . \tag{4.9}$$

The adaptive (self-tuning) algorithm for unknown parameters [A1]

(i) estimates the parameters (using, e.g., least squares) and

(ii) uses same controller with estimated parameters in place of the true ones.

The STURE is passively adaptive and belongs to the F class. The optimal policy is of the CL type since the control has a dual effect. Several more general versions are available as well as convergence results [L1]. A number of successful applications to practical problems have been reported [A2].

4.3 Multiple-Model Weighted (Partitioned) Adaptive Control

The system is

$$x_{k+1} = F(\theta)x_k + G(\theta)u_k + v_k \tag{4.10}$$

$$y_k = H(\theta)x_k + w_k , \tag{4.11}$$

with cost

$$C = x_N'Q_N x_N + \sum_{k=0}^{N-1} x_k'Q_k x_k + u_k'R_k u_k . \tag{4.12}$$

The parameter vector belongs to a *finite set* (*"set of models"*). The initial information is

$$P[\theta = \theta_i | I^0] = p_{i,0} \qquad i = 1,\ldots,M . \tag{4.13}$$

For known parameters, the optimal control is

$$u_k^*(\theta) = - L_k(\theta)\hat{x}_{k|k}(\theta) . \tag{4.14}$$

The controller in this approach [D1] is a weighted sum of the model-optimal controllers

$$u_k^{MMAC} = \sum_{i=1}^{M} u_k^*(\theta_i)P[\theta = \theta_i | I^k] . \tag{4.15}$$

Note that this is not optimal and only passively adaptive. The recursive updating of the parameter pdf is done using Bayes' rule as follows:

$$p_{i,k} = P[\theta = \theta_i | I^k] = \frac{p(y_k | I^{k-1}, \theta_i) p_{i,k-1}}{\sum_{j=1}^{M} p(y_k | I^{k-1}, \theta_j) p_{j,k-1}} .$$

4.4 A Closed-Loop (Dual) Control Algorithm

The stochastic dynamic programming equation

$$J^*(k, I^k) = \min_u E\{C_k[x(k), u(k)] + J^*(k+1, |I^{k+1}) | I^k\} \tag{4.16}$$

is approximated in this approach as follows [T1, T2, B3].

- The full information vector I^k is replaced by an approximate information state

$$P^k = \{\hat{x}(k|k), \Sigma(k|k)\} . \tag{4.17}$$

  The vector x is the "proper" state augmented by the system's unknown parameters, if any. An estimator like the EKF can be used to generate this information state.

The following search procedure is used to find the control at time k:

- An arbitrary control u_k is assumed.
- This control yields a predicted state $\hat{x}[k+1|k;u(k)] \triangleq x_0(k+1)$.
- The resulting predicted state $x_0(k+1)$ is taken as the initial condition of a "nominal" trajectory $x_0(j)$, $j = k+1,\ldots,N$ generated with a sequence of nominal controls $u_0(j)$, $j = k+1,\ldots,N$.
- A perturbation analysis via second order expansion is carried out about the nominal trajectory to capture the stochastic effects.

- The expected cost corresponding to this control is then evaluated using a set of recursions.
- The procedure is repeated to find the value of the control that yields the minimum of the expected cost.

The resulting control depends on

- Current estimate of the (augmented) state.
- Current state uncertainty.
- Future state uncertainties as anticipated: The covariance of the state is precomputed along the nominal trajectory via EKF. Note that this nominal trajectory depends on the current control u(k). Thus, if the control has the dual effect, its effect on the quality of future information, $\Sigma(j|j)$, $j > k$, is automatically incorporated in the decision procedure.

This algorithm has been used for

- Linear systems with unknown parameters (actively adaptive control).
- Nonlinear systems (with no unknown parameters) where the control can enhance the estimation.

The algorithm consists of the following

$$u^{CL}(k) = \arg\min J^{CL}(k) \; . \tag{4.18}$$

A very useful decomposition of the (closed-loop) approximation of the optimal) cost can be obtained [B4, B5]:

$$J^{CL}(k) = J_D(k) + J_C(k) + J_P(k) \; . \tag{4.19}$$

The deterministic component of the cost is

$$J_D(k) \triangleq \phi_k[u(k)] + C_0(k+1) + \gamma_0(k+1) \tag{4.20}$$

and the stochastic components of the cost are

$$J_C(k) \triangleq \frac{1}{2}\,tr[K_0(k+1)\Sigma(k+1|k)]$$

$$+\frac{1}{2}\sum_{j=K=1}^{N-1} tr[K_0(j+1)V_v(j)] \qquad (4.21)$$

$$J_P(k) \triangleq \frac{1}{2}\sum_{j=k+1}^{N-1} tr[A_0(j)\Sigma_0(j|j)] \qquad (4.22)$$

where

K_0, A_0 - obtained from some recursions.

Σ_0 - covariance of the augmented state evaluated along the nominal trajectory.

V_v - covariance of the process noise.

The deterministic component consists of

(i) $\phi_k[u(k)]$ - cost of control at time k.

(ii) $C_0(k+1)$ - cost incurred along the nominal trajectory from k+1 to N; all uncertainties are ignored (HCE).

The caution component consists of

(i) $\frac{1}{2}\,tr[K_0(k+1)\Sigma(k+1|k]$ - cost due to the uncertainty in the initial condition $x_0(k+1)$ of the nominal trajectory. This is a mapping of the current uncertainty - its effect on the cost.

(ii) $\frac{1}{2}\sum_{j=k+1}^{N-1} tr[K_0(j+1)V_v(j)]$ - cost due to the disturbances in the dynamic equation (process noise).

The caution component represents the effects of the existing uncertainties on the cost. The weighting are, however, depending on the choice of the current control u(k).

The probing component is

$\frac{1}{2}\sum_{j=k+1}^{N-1} tr[A_0(j)\Sigma_0(j|j)]$ - weighted sum of the <u>future</u> uncertainties (state covariances). These uncertainties depend on the current

control if it has the dual effect. The weighting matrix A_0 reflects the "value of future information."

Remarks:

1. The caution component tends (in general, but not always) to reduce the value of the control: larger control values might increase the effect of the uncertainties on the cost. The control will be "cautious" due to the uncertainty.
2. In some problems it pays off for the control to probe in order to reduce the uncertainties about the system.
3. A compromise between the control action, the need for caution and the desirability of probing has to be made.
4. Probing and caution are usually, but not always, conflicting.

Based on the relative magnitude of the three cost components one has three major classes of stochastic control problems: If the uncertainty dominates the problem then one can distinguish two cases

1. The caution component (J_C) dominates. Then, since this is "uncontrollable" uncertainty, one has a highly uncertain model which cannot be improved in the course of the control period.
2. The probing component (J_P) dominates. Then, with the dual effect of the control one can reduce the uncertainty of the model - thus the model, while uncertain at the beginning, might prove to be ultimately adequate for the control problem under consideration.

A third case occurs when:

3. The deterministic component of the cost, J_D, dominates: then the parameter uncertainties are of no significant consequence. This is the most desirable situation because then we can use CE (least expensive) control algorithm with good performance. However, only the stochastic control approach can tell us that fact.

4.5 Dual Control for a Missile Guidance Problem

The missile is modeled by point mass kinematics with lateral acceleration

$$\ddot{\xi} = -\dot{\eta}\,u \qquad \ddot{\eta} = \dot{\xi}\,u \tag{4.23}$$

which leads to the following nonlinear stochastic plant equation

$$\dot{x}(t) = f[x(t)u(t)] + v \tag{4.24}$$

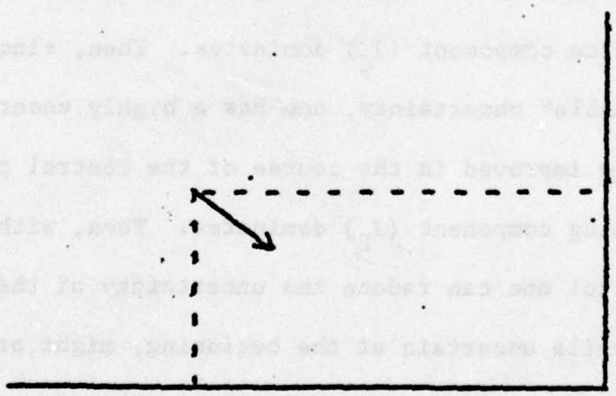
where

$$f[x(t)u(t)] = [x_2(t)\ \ -x_4(t)u(t)\ \ x_4(t)\ \ x_3(t)u(t)]' . \tag{4.25}$$

The measurement consists of angle only

$$y(t_k) = \tan^{-1}\frac{x_3(t_k)}{x_1(t_k)} + w(t_k) . \tag{4.26}$$

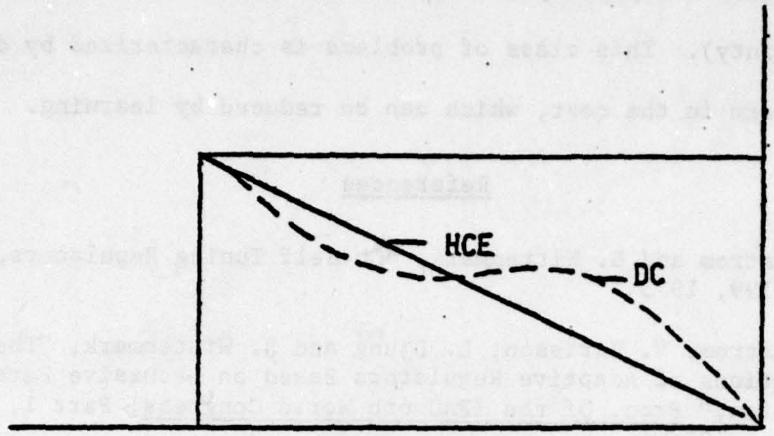
The goal of the guidance is to hit the origin with noisy initial information about own initial location and velocity.



Remarks:

1. There are no unknown parameters in this problem.
2. The main problem is the (nonlinear) estimation of the state, in particular the range to the target.
3. Under straight flight conditions the on-board angle-only sensor will provide little information about the range.

The trajectories with HCE vs. dual control [T3] turned out to be as illustrated below:



The dual controller suggests deviating on both sides to obtain range information from the angle-only sensor by varying the geometry of the problem during the flight.

Once this observation is made, an off-line optimization algorithm has been used to obtain the divert maneuver that minimizes the terminal miss distance [C1].

Usefulness of these results is in the following:

1. Improvement of miss distance for given sensor accuracy, or
2. Lowering of sensor accuracy requirements for given miss distance.

5. Conclusions

Wonham at the 1968 JACC stated the following: In the case of (stochastic) feedback controls the general conclusion is that only marginal improvement can be obtained (over a controller ignoring the stochastic features), unless the disturbance level is very high; in this case the fractional improvement may be large but the system is useless anyway.

Recent results show that in some problems adaptation and, in particular, active adaptation can yield performance close to the lower bound (when there is no uncertainty). This class of problems is characterized by dominance of the probing term in the cost, which can be reduced by learning.

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III. INDIVIDUAL CONTRIBUTIONS

COMMENTS ON ADAPTIVE AND ROBUST CONTROL

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ADAPTIVE CONTROL

I believe the following basic questions must be answered when a flight control application of adaptive control is contemplated.

> Is the synthesis technology for adaptive control adequately developed?
>
> Is adaptive control necessary or highly advantageous?

These questions can be refined. For example, "adequately developed" can be quantified in terms such as the skill required of the designer and the cost of the synthesis in computer and engineering hours. Similarly, "highly advantageous" can be quantified and "necessary" can be qualified in terms of system performance and system cost.

In the NASA flight control research program in digital fly-by-wire technology it was demonstrated that some adaptive control technology is adequately developed for synthesizing an adaptive command augmentation system for an F-8C aircraft. But the performance requirements for the F-8C are readily met with air-data-scheduled (nonadaptive) control laws. Thus the major benefit provided by adaptive control is that the need for air-data sensors could be eliminated which would be advantageous from redundancy considerations. I believe that this example is typical and that similar answers would apply to future high performance aircraft.

A flight control application in which adaptive control seems to be necessary or highly advantageous is the active control of wing-store flutter. The motivation for this application is that fighter aircraft are required to carry many different combinations of external stores to perform a variety of

missions. Wing mounting of these stores can cause significant reductions in wing flutter speeds and can give rise to different flutter modes with significantly different frequencies. Passive means to accommodate these situations result in structural modifications or the imposition of speed placards. These passive methods generally reduce aircraft performance. Thus, active control of flutter is a promising alternative especially with the development of highly reliable fly-by-wire technology. To accommodate the variety of possible flutter modes involved in wing/store flutter, an adaptive capability is highly desirable. Thus, there is an affirmative answer to the second question.

With regard to the first question, this application raises two issues.

Issue 1: The synthesis technology must provide stabilization of an unstable system with a rapid speed of response. This requirement arises from the possibility that a change in store configuration can cause the aircraft to suddenly have an unstable flexure mode corresponding to the new store configuration. To add to the challenge, it is, of course, possible that the adaptive controller was actively stabilizing the flutter mode corresponding to the configuration before the change. This requires the adaptive controller to adapt from one unstable oscillatory mode to another rapidly enough to prevent structural damage to the aircraft.

Issue 2: The synthesis technology must apply to infinite dimensional dynamics which are crudely approximated by finite dimensional linear systems. This issue is common to all flexure control problems but especially important for ones involving high frequency aero-elastic dynamics. The structural and aerodynamic models used in synthesis are increasingly inaccurate with increasing frequency.

A second possible application that is currently being contemplated is the adaptive control of the Space Shuttle Orbiter. Here the motivation is that there will be significant differences in payloads with attendant differences in center of mass, moments of inertia, and changes in flexure characteristics. A controller tailored to each payload could be designed, but adaptive control would be very convenient. The demands on the technology for this application appear to be much less severe than the wing/store flutter application. However, since the motivation for adaptive control is convenience, the monetary cost of designing such an adaptive control must be low to be of significant benefit over tailored designs.

ROBUST CONTROL

Probably each participant has his/her own definition of robust control. I believe the two major robustness properties are performance invariance and stability invariance. Performance invariance relates to disturbance rejection and command following and generally implies that the return difference matrix is large. Stability invariance generally implies that the return difference matrix is attenuated sufficiently at high frequencies to provide margins consistent with expected uncertainty levels.

It is my opinion that every concept of robust control should give consideration to each of these properties. Furthermore, adaptive synthesis techniques used to treat parameter uncertainty and provide performance invariance should not ignore robustness with respect to unmodeled dynamics and nonlinearities.

COMMENTS ON AIRCRAFT CONTROL PROBLEMS

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The Control Analysis Group was organized within the AFFDL a year and a half ago to further research and applications in advanced control analysis methods. We initiated new efforts and carried on several existing efforts in this area. A quick rundown of these efforts follow. In addition, a statement is given relative to a real world problem area that may show significant payoff through the application of adaptive control concepts.

EFFORTS:

Recently a contract was awarded through our office to Dr. Mehra of Scientific Systems, Inc. using basic research funding from AFOSR. This effort is entitled Basic Research in Digital Stochastic Model Algorithmic Control.

Two efforts in the area of analysis methods for digital control systems have been funded using basic research funding. Both efforts were awarded to Dr. Whitbeck of Systems Technology, Inc. The first effort, entitled Analysis of Digital Flight Control Systems with Flying Qualities Application, has resulted in AFFDL TR-78-115. The second effort pursuing direct digital design methods was recently awarded. It is entitled Digital Control Systems Synthesis Using Multiple Order Sampling.

The Control Analysis Group is also involved with Major Gary Reid of AFIT in the sponsorship of Masters level thesis efforts applying Dr. Mehra's work on Model Algorithmic Control Application to B-52 flutter mode control problems.

In-house work is also being accomplished relative to the formulations of a definitive statement of integrated control concepts which encompass six-degree-

of-freedom digital flight control, fault tolerant design, microprocessors, and parallel processing. This effort is largely a planning effort at this stage, but will focus and harmonize a significant group of existing and planned efforts related to integrated control within our division.

PROBLEM AREA FOR ADAPTIVE CONTROL:

The near stall flight regime is fraught with highly nonlinear changes in dominant aerodynamic coefficients such as $C_{n\beta}$, $C_{\ell\beta}$, and $C_{m\alpha}$. Linear analysis and synthesis methods are normally used for control system design which optimize system dynamics at lower angles of attack where the system model behaves in a more nearly linear fashion. Normally the designer then evaluates what he has at higher angles of attack through nonlinear simulation. If real problems are apparent, the designer either limits the aircraft to lower angles of attack through placards placed upon that flight regime, or he may develop a limiting type of flight control system that inhibits the aircraft from entering the dangerous flight regimes existing at higher angles of attack. It is apparent that if adaptive control systems could be designed to follow the vehicle dynamics in the near stall flight regime, and if sufficient control power could be generated to provide the necessary level of control, then real improvements in safety of flight, as well as increased usable maneuverability, could be achieved. Controlled flight at high angles of attack is a real problem.

ON ROBUSTNESS

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The state of the art in adaptive control is such that in the majority of situations truly optimal dual control solutions are computationally infeasible. Consequently, in implementing practical adaptive control techniques it is almost inevitable that one must make simplifying assumptions or approximations--e.g., that parameters vary slowly or that the system operating plant changes slowly. While in practical situations one frequently finds that these assumptions and approximations are quite reasonable, one ultimately must resort to simulation or to stability theoretic techniques to establish their validity.

Robustness in the context of control engineering is the tolerance of a control design to uncertainty and imprecision in modeling--including imprecision that is intentionally introduced in the form of simplifying assumptions or approximations. The significance of robustness in the area of adaptive control is twofold. First, practical adaptive control designs must have a certain degree of robustness if simplifying assumptions about slowly varying parameters, operating points and so forth are to be valid. Second, robustness is a significant issue in adaptive control because of its implications about when adaptive behavior is even necessary in a control design: if one can design a control law under the extreme simplifying assumption that parameters and operating points do not vary at all and if that control law is sufficiently robust then adaptive control is plainly unnecessary.

Efforts to quantitatively characterize robustness have inevitably been related to stability and sensitivity theory, state-space (Lyapunov) and input-output techniques both being effective. However, because the quantitative

amount of robustness at any particular node in an interconnected system is directly related to the return-difference at the node (an input-output relation), it is my present opinion that input-output methods provide a more direct and conceptually simple method to address robustness issues in control, the role of the state-space being primarily in the representation of input-output relations for computational purposes. My recent research in the area of robustness reflects this view. Robustness of the property of stability is addressed in an input-output setting in refs. 1-4, 6, 7. Robustness of system response (i.e., sensitivity) is addressed in ref. 5. Reference 4 contains a result which is especially relevant to adaptive control: one can substitute nondivergent estimates (e.g., estimates from a globally "incrementally" stable nonlinear observer) for true values in a control system without inducing instability. Nondivergence is a property of the estimator itself and is not control law dependent. So, insofar as stability is concerned the design of an adaptive controller can with complete rigor be separated into two parts, a parameter estimator and a parameter-dependent control law.

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NONROBUSTNESS AND BIFURCATION*

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Bifurcation phenomena in nonlinear systems is known to cause extreme sensitivity of system behavior to parameter variations. We consider the specific case of Linear-Quadratic-Gaussian (LQG) control. The nonrobustness of LQG is related to the bifurcation behavior of the Riccati equation. Specifically, the null solution of the Kalman filter Riccati equation for the zero process noise case bifurcates when an eigenvalue of the system crosses the imaginary axis from left half to right half plane (transition from a stable to an unstable system). Another bifurcation of the Riccati equation occurs for transitions from minimum phase to nonminimum phase characteristics, i.e., when one of the zeros of the system crosses the imaginary axis from the left to the right half plane. The latter case occurs when the measurement noise covariance is zero. The above two bifurcations of the Riccati equation explain the difference in robustness properties of the LQ design versus LQG design. The latter loses robustness as the open loop system dynamics become unstable or nonminimum phase.

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A NEW FORMULATION OF THE MULTIVARIABLE ROBUST SERVOMECHANISM PROBLEM

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The design of multivariable control systems to provide zero steady-state system error in the presence of non-decaying disturbances and non-decaying reference signals and in spite of perturbations to system parameters has been formulated and solved by Davison in a series of papers[1,2,3] and, using geometric methods, by Francis,[4,5] Wonham[5,6] and Pearson[6]. The latter authors present the essential structure of the solution as the "internal model principle". The version of the problem to be studied here may be described by the following equations

state	$\dot{x} = Fx + Gu + G_1 w$	n_sx1	(1)
output	$y = Hx + Ju + J_1 w$	n_ox1	(2)
error	$e = y - r$	n_ox1	(3)
reference	$r^{(P)} = \sum_{i=1}^{P} \alpha_i r^{(P-i)}$; r(0) unknown	n_ox1	(4)
disturbance	$w^{(P)} = \sum_{i=1}^{P} \alpha_i w^{(P-i)}$; w(0) unknown	n_dx1	(5)
control	$u = k(y,e)$	n_cx1	

In (4) and (5), the α_i are real scalars. If $\alpha(s) = s^P - \sum_{i=1}^{P} \alpha_i s^{P-i}$, then $\alpha(s)$ is the polynomial of lowest degree for which r and w satisfy the corresponding differential equation.

The problem is to design a control law k(y,e) to provide <u>regulation</u>, which is to say that the error, e, tends to zero as time gets large even (especially!) if the α_i are such that r and w grow without bound with time.

The control must also be structurally stable or robust in the sense that regulation occurs in the presence of perturbations of the greatest possible number of system parameters.

Part I. Regulation

The state of the dynamical system described by (1), (4), and (5) is of dimension $n_s + P \cdot n_o + P \cdot n_d$. Since the control signal does not appear in (4) or (5) it is clear that this system is not controllable. However the task is to control the error, e(t). Since the error is a linear function of the overall state, one should be able to find a coordinate system within which the error is directly a coordinate of the state. The problem of error control may then be expressed in the well known terms of state control.

Since the error contains the reference input and that signal satisfies differential equation (4) which has P^{th} order derivatives, an equation in the error of similar structure is considered.

$$e^{(P)} - \sum_{i=1}^{P} \beta_i e^{(P-i)} = y^{(P)} - \sum_{i=1}^{P} \beta_i y^{(P-i)}$$

$$- r^{(P)} + \sum_{i=1}^{P} \beta_i r^{(P-i)} \quad . \qquad (6)$$

Substituting (4) for $r^{(P)}$ in (6)

$$e^{(P)} - \sum_{i=1}^{P} \beta_i e^{(P-i)} = y^{(P)} - \sum_{i=1}^{P} \beta_i y^{(P-i)}$$

$$+ \sum_{i=1}^{P} (\beta_i - \alpha_i) r^{(P-i)} \quad . \qquad (7)$$

It is noticed that the (uncontrollable) reference input will vanish from the error equation if and only if $\beta_i = \alpha_i$. If this selection is made and y is expanded from (2), (7) becomes

$$e^{(P)} - \sum_{i=1}^{P} \alpha_i e^{(P-i)} = H[x^{(P)} - \sum_{i=1}^{P} \alpha_i x^{(P-i)}]$$

$$+ J[u^{(P)} - \sum_{i=1}^{P} \alpha_i u^{(P-i)}]$$

$$+ J_1[w^{(P)} - \sum_{i=1}^{P} \alpha_i w^{(P-i)}] . \tag{8}$$

The reason for selecting $\alpha(s)$ to describe the dynamics of both r and w becomes clear: the disturbance vanishes from (8) because of (5), and there is a possibility of controlling the error state from the input u. To complete the description the plant state x is replaced by ξ defined as

$$\xi = x^{(P)} - \sum_{i=1}^{P} \alpha_i x^{(P-i)} \tag{9}$$

and the control is replaced by

$$\mu = u^{(P)} - \sum_{i=1}^{P} \alpha_i u^{(P-i)} \tag{10}$$

then (8) becomes

$$e^{(P)} - \sum_{i=1}^{P} \alpha_i e^{(P-i)} = H\xi + J\mu . \tag{11}$$

The state equation for ξ is given by

$$\dot{\xi} = x^{(P+1)} - \sum_{i=1}^{P} \alpha_i x^{(P-i+1)}$$

$$= Fx^{(P)} + Gu^{(P)} + G_1 w^{(P)} \tag{12}$$

$$- \sum_{i=1}^{P} \alpha_i F x^{(P-i)} - \sum_{i=1}^{P} \alpha_i G u^{(P-i)} - \sum_{i=1}^{P} \alpha_i G_1 w^{(P-i)} .$$

Since the α_i are scalars, $\alpha_i F = F\alpha_i$ and (12) may reduce to

$$\dot{\xi} = F\xi + G\mu . \tag{13}$$

Equations (11), (13), and (5) now describe the overall system state and the portion containing the error is given by (11) and (13). In state variable form these are

$$\dot{z} = Az + B\mu \tag{14}$$

where† $Z' = [e', \dot{e}', \ldots, e^{(P-1)}; \xi']$ and

$$A = \begin{bmatrix} 0 & I & 0 & \cdots & 0 & 0 \\ 0 & 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & \cdots & I & 0 \\ \alpha_1 I & \alpha_2 I & & \cdots & \alpha_p I & H \\ 0 & 0 & & \cdots & 0 & F \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ J \\ G \end{bmatrix} \tag{15}$$

The error can be forced to zero if and only if (A,B) are stabilizable and can be given arbitrary dynamics if (A,B) are controllable. Only the conditions for controllability are presented.

The matrices (A,B) are controllable if and only if[7]

$$\text{rank } (sI - A \vdots B) = n_s + P \cdot n_o . \tag{16}$$

By elementary row operations, (16) is reduced to the condition

$$\text{rank} \begin{bmatrix} sI & -I & 0 & \cdots & 0 & 0 & 0 \\ 0 & sI & -I & \cdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & & -I & 0 & 0 \\ 0 & 0 & 0 & \cdots & \frac{\alpha(s)}{s^{P-1}} I & -H & J \\ 0 & 0 & 0 & \cdots & 0 & sI-F & G \end{bmatrix} = n_s + P \cdot n_o \tag{17}$$

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†Z' equals the transpose of Z.

Since the matrix in (17) has $n_s + P \cdot n_o$ rows, all the rows must be independent and especially the last n_s rows must be independent for all s. These are the rows of $[sI - F{:}G]$ and requires that (F,G), the plant, be controllable. If we let $s = \lambda_i$ when $\alpha(\lambda_i) = 0$ (λ_i is a characteristic value of the dynamic systems which produce r and w) it is obvious that the final condition for controllability is given by

$$\text{rank} \begin{bmatrix} -H & J \\ \lambda_i - F & G \end{bmatrix} = n_s + n_o . \qquad (18)$$

Since this matrix has $n_s + n_c$ columns, and the rank is no more than the minimum of $(n_s + n_c;\ n_s + n_o)$, it is required that $n_c \geq n_o$, or that there must be as many controls as there are outputs. A value of λ_i for which the rank of (18) is less than $n_s + n_o$ is a zero (7,8) of the plant. In summary, the error state is controllable if and only if

(i) F,G is controllable

(ii) $n_c \geq n_o$ (19)

(iii) H,F,G,J has no zeros at λ_i for which $\alpha(\lambda_i) = 0$.

If the conditions of (19) are met, then there exists a control law $\mu = -Kz$ so that the error-state system of (14) has an arbitrary characteristic equation. The control gain K may be computed by any method such as optimal quadratic loss, pole assignment, or frequency response synthesis.

Part II. Implementation

Once the control law is designed, it is necessary to implement it by constructing the plant control u, from the system error e and the plant state, x, if it is available, or from an estimate of the state if only the output is sensed. For this development, the control gain is first partitioned according to the elements of the state z as

$$\mu = -K_p e - K_{P-1}\dot{e} - \dots - K_1 e^{(P-1)} - K_o \xi \ . \tag{20}$$

If (9) and (10) are substituted into (20) for ξ and μ, the controller equations become

$$u^{(P)} - \sum_{i=1}^{P} \alpha_i u^{(P-i)} = -\sum_{i=1}^{P} K_i e^{(P-i)} - K_o\{x^{(P)} - \sum_{i=1}^{P} \alpha_i x^{(P-i)}\} . \tag{21}$$

Recognizing that u and x enter (21) by equations with identical coefficients, (21) can be written as

$$(u + K_o x)^{(P)} = \sum_{i=1}^{P} \alpha_i (u + K_o x)^{(P-i)} - \sum_{i=1}^{P} K_i e^{(P-i)} \ . \tag{22}$$

It is noticed immediately that (22) represents the servocompensator of Davison[1,2,3] and the internal model of Francis and Wonham.[5] As an aside from the perspective of classical control, it is well known that system error in a unity feedback topology is reduced by the loop gain. The requirement that the error be zero at frequencies λ_i such that $\alpha(\lambda_i) = 0$ would be expected to call for infinite gain, i.e., poles, at λ_i.

However, the control of (22) assumes that all the plant states are available. If only the error e and the output y are measured, it is possible to estimate x by a linear observer.[9†] A suitable (but not minimal order) estimator equation is[††]

$$\dot{\hat{x}} = F\hat{x} + Gu - L(y - Ju - H\hat{x}) \ . \tag{23}$$

The estimate error associated with (23) is

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† One may also control (14) by a dynamic compensator, of course.[10]

†† The estimator in (23) can use any measurement of the state for correction in place of the system output y. Some authors identify y_m as the signal to be measured to distinguish it from the output to be controlled.

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = Fx + Gu + G_1w - F\hat{x} - Gu$$

$$- L(Hx + J_1w - H\hat{x})$$

$$\dot{\tilde{x}} = (F-LH)\tilde{x} + (G_1 - LJ_1)w \quad . \tag{24}$$

If F,H are observable, then L may be selected to give F-LH an arbitrary characteristic equation. However, since w is a possibly growing signal, it is likely that $\tilde{x}$ will also grow in time. It is necessary to demonstrate that use of $\hat{x}$ from (23) in place of x in the controller (21) will not cause the system error, e, to fail to tend to zero. If x is replaced by $\hat{x}$ in (21) there results the modified controller equation

$$u^{(P)} - \sum_{i=1}^{P} \alpha_i u^{(P-i)} = - \sum_{i=1}^{P} K_i e^{(P-i)} - K_o\{\hat{x}^{(P)} - \sum_{i=1}^{P} \alpha_i \hat{x}^{(P-i)}\}. \tag{25}$$

Now $\hat{x} = x-\tilde{x}$ and $\hat{x}^{(k)} = x^{(k)} - \tilde{x}^{(k)}$. Therefore (25) is equivalent to

$$u^{(P)} - \sum_{i=1}^{P} \alpha_i u^{(P-i)} = - \sum_{i=1}^{P} K_i e^{(P-i)} - \{K_o x^{(P)} - \sum_{i=1}^{P} \alpha_i x^{(P-i)}\}$$

$$+ K_o\{\tilde{x}^{(P)} - \sum_{i=1}^{P} \alpha_i \tilde{x}^{(P-i)}\} . \tag{26}$$

If (24) is used to evaluate the last term in (26), one obtains

$$\tilde{x}^{(P+1)} - \sum_{i=1}^{P} \alpha_i \tilde{x}^{(P-i+1)} = (F-LH)[\tilde{x}^{(P)} - \sum_{i=1}^{P} \alpha_i \tilde{x}^{(P-i)}] . \tag{27}$$

Defining $\eta(t) = \tilde{x}^{(P)} - \sum_{i=1}^{P} \alpha_i \tilde{x}^{(P-i)}$, (27) is

$$\dot{\eta} = (F-LH)\eta \tag{28}$$

and (26) is reduced to

$$u^{(P)} - \sum_{i=1}^{P} \alpha_i u^{(P-i)} = - \sum_{i=1}^{P} K_i e^{(P-i)}$$

$$- K_o\{x^{(P)} - \sum_{i=1}^{P} \alpha_i x^{(P-i)}\} + K_o \eta(t). \quad (29)$$

Thus the effect of using the estimated state is to cause the control to have added a term $K_o \eta$ which is the output of (28), a dynamic system with a characteristic equation arbitrarily selected by choice of L if F,H is observable.

Part III. Robustness

From the nature of the pole-assignment problem, the state Z in (14) will tend to zero for all perturbations in the system parameters which leave A-BK, and F-LH stable. However, if the elements of H, J, or J_1 in (2) are changed, then the system will force the perturbed output to be equal to the reference signal. Thus the system forces the output of the sensors to track the reference signal and is not robust with respect to perturbations in the sensor parameters. The second limitation on robustness derives from (7) where it was required that $\beta_i = \alpha_i$ if the reference and disturbance signals are to vanish from the error state. The solution is thus not robust with respect to the selection of the α_i in the control law (22). From the point of view of the implementation, one can say that the values of α_i implemented define the class of reference and disturbance signals for which the error certainly goes to zero. If the system is subjected to signals which fail to satisfy (4) or (5) with α_i as implemented in (22), then the error cannot be guaranteed to tend to zero. An important special case with roots deep in classical control occurs when all $\alpha_i = 0$. The signals defined by (4) and (5) are then polynomials in time and the controller can typically be implemented as a chain of integrators with great accuracy.

Part IV. Examples

Example 1 illustrates the introduction of integral control to a first order plant.

$$\dot{x} = -2x + u + w$$

$$y = x$$

$$e = y - r$$

$$\dot{r} = 0$$

$$\dot{w} = 0$$

The error state matrices are, from (15)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

A control gain which gives the characteristic polynomial $(s+1)^2+4$ is

$$K = [8 \quad 2] .$$

The control law (22) is

$$\dot{u} = -8e - 2\dot{x}$$

or

$$u = -8 \int^{t} e \, dt - 2x . \qquad (30)$$

Equation (30) shows explicitly the integral control on the system error.

Example 2. A more complex example, but still single input single output is motivated by a servomechanism to follow the data track on a computer disk memory system. Because the data track is not exactly a centered circle, the radial servo must follow a ("runout") sinusoid input of radian frequency ω_o. The (normalized) parameters are

$$F = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \; ; \; G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \; ; \; H = [1 \;\; 0] \; ;$$

$$J = 0 \; ; \; J_1 - 0 \; ; \; G_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\ddot{r} = -\omega^2 r \; .$$

The error state matrix is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega^2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} ; \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} .$$

The characteristic equation of A-BK is

$$s^4 + K_{02}s^3 + (\omega^2 + K_{01})s^2 + (K_1 + \omega^2 K_{02})s + K_2 + \omega^2 K_{01} = 0 \qquad (31)$$

from which the gain may be selected if pole assignment is satisfactory for the design. Since x_1 is the output of the plant, it is available for feedback. An estimator for x_2 could be described by the equation ($\dot{x}_1 = x_2$ is used as the "measurement" of x_2 for the estimator design):

$$\dot{\hat{x}}_2 = -\hat{x} + u + L(\dot{x}_1 - \hat{x}_2) \; . \qquad (32)$$

The estimator error equation is

$$\dot{\tilde{x}}_2 = -(+1+L)\tilde{x}_2 \; . \qquad (33)$$

The estimator gain may be selected from (33). The "servocompensator" consisting of the oscillator with frequency ω is clearly seen.

Conclusions: The robust servomechanism problem has been formulated in the error space which allows an alternate derivation of most of the known results using only the theory of controllability. It is believed that this formulation is in some ways simpler than previous presentations.

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A UNIFICATION OF ADAPTIVE AND ROBUST CONTROL CONCEPTS*

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The naivety of the idea of changing the controller as plant parameters vary from which the adaptive control concepts are derived is soley responsible for the multitude of adaptive control structures of various degrees of sophistication ranging from the self-optimizing types of the early days to the self-tuning type and the model reference adaptive type of today. The design goal of these adaptive control schemes is to assure that plant performance specifications (such as stability) are met under all foreseeable plant parameter variations often caused by changes in operating conditions. This same design goal is shared by control engineers whose concern is essentially that if a fixed gain feedback controller is designed for the nominal plant whether it remains effective as the plant parameter changes. The robust control concept is the outcome of this concern: design fixed gain feedback controller that are robust in the sense that plant performances remain satisfactory when the plant parameters vary arbitrarily within a certain set.

Robust control and adaptive control concepts are often considered to be distinctively different. It is customary to associate immediately linear fixed gain feedback controller structures with robust control concepts and complex nonlinear controller structures with adaptive control concepts. The distinction between robust control and adaptive control concepts becomes superficial when recently a model reference adaptive control problem is solved using the theory of variable structure systems and sliding mode.[1] The resulting adaptive

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controller is a variable structure feedback controller[2] and indeed possesses an inherent adaptation mechanism. The success of this adaptive control solution can be accredited to the achievement of parameter insensitivity when this adaptive control system is in sliding mode. In the robust control framework, variable structure feedback has also been shown to be effective in preserving plant performances under plant parameter variations because of the inherent insensitivity properties of variable structure feedback systems. Variable structure control in fact can be viewed as either a robust control or adaptive control concept. In the robust control context, it is known that variable structure feedback controller has the same insensitivity property of high fixed gain linear feedback controller. In the adaptive control context, it is indeed a controller with changing parameters and the controller structure is varied in a true adaptive sense.

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[1] K. K. Young, "Design of Variable Structure Model - Following Control Systems," IEEE Trans. Automatic Control, Vol. AC-23, pp. 1079-1085, 1978.

[2] A variable structure feedback controller in this case is a linear feedback controller whose feedback elements are discontinuous on some switching hyperplanes.

SOME ASPECTS OF INSENSITIVE AND ADAPTIVE CONTROL

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1. ON INSENSITIVE CONTROL VERSUS MRAC

It is well known, that feedback can reduce the effect of plant parameter variations. See, for example, the conditional feedback structure of Fig. 1. The error signal Δy is fed back to make the transfer behaviour $\{u_c \to y\} \approx \{u_c \to y_N\}$ for all operating conditions.

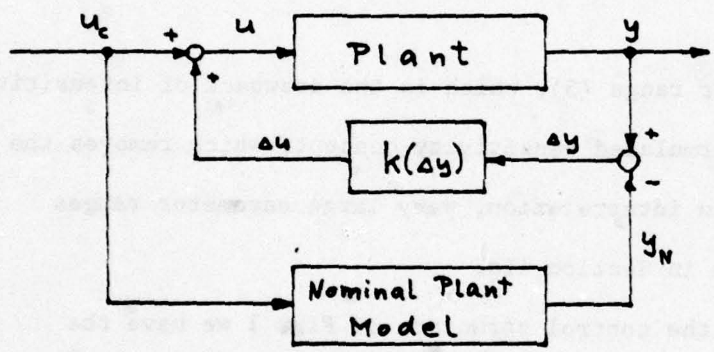

Fig. 1
Comparison sensitivity reduction control structure.

In case of ideal compensation we would have

$$\Delta y \equiv 0 \quad \text{but} \quad \Delta u \not\equiv 0 \quad \Rightarrow \text{"infinite loop gain"}$$

i.e. the sensitivity problem as formulated above has a strong <u>low sensitivity - high loop gain</u> interrelation.

Consequently, in such an insensitive design, which is primarily concerned with linear feedback laws, the following is essential:

(1) Use of <u>realizable feedback gains</u> only, which are often much lower than those necessary for a desirable sensitivity reduction, i.e. a practically feasible compromise must be reached.

(2) The <u>disturbance rejection behaviour</u> as well as the disturbance sensitivity depend on the same feedback and must be taken into account in this compromise.

(3) The <u>range of parameter variations</u>, which can be covered also depends on the feedback gains and, due to (1), may be not very large.

(4) <u>Linearity</u> of both the plant and the feedback law substantially simplify the design and allow easy closed loop system analysis.

It is the linearity (4) which makes insensitive control particularly attractive for practical applications. Here a lot of experience is available and no such basic problems comparable to the stability problem in adaptive control exist.

It is the limited parameter range (3), which is the drawback of insensitive control. However, using a reformulated sensitivity concept, which removes the low sensitivity - high loop gain interrelation, very large parameter ranges can be covered as will be shown in Section 11a.

As a natural extension of the control structure of Fig. 1 we have the typical MRAC structure of Fig. 2.

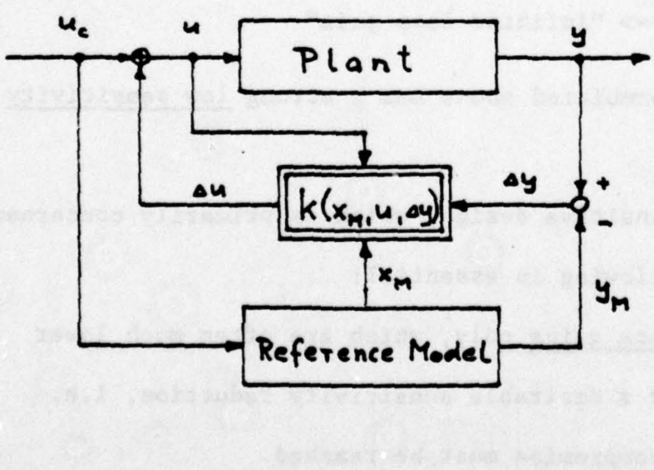

Fig. 2
Model reference adaptive control structure

Again the control law is to make the transfer behaviour $\{u_c \rightarrow y\} = \{u_c \rightarrow y_M\}$. But now the control is modified to be nonlinear, where the nonlinearity is the outcome of the idea to adapt the control in order that $\{u_c \rightarrow y\}$ matches $\{u_c \rightarrow y_M\}$ asymptotically.

The main advantages of MRAC over insensitive control are:

(5) Zero sensitivity is obtained asymptotically where only

(6) finite feedback gains are involved.

(7) Arbitrarily large parameter ranges can be covered with asymptotic zero sensitivity.

These advantages are associated with the following substantial shortcomings:

(8) The control system is highly nonlinear, which complicates the design and, in particular, the closed loop system analysis.

(9) To prove global stability requires a minimum phase plant as well as a priori knowledge such as the relative degree of the plant transfer function or the sign of its static gain.

(10) The disturbance rejection behaviour is essentially unresolved both from the (deterministic) stability and the regulation accuracy point of view.

Nevertheless MRAC does work in practice and major progress in proving global stability of such schemes is being made right now. Obviously a MRAC design is subject to practical constraints imposed on the adaptive gains and hence the speed of adaptation as well as to the disturbance rejection behaviour, similar to the insensitive design (1) - (2).

At this point a very strange facet of adaptive control is worth mentioning, to illustrate the basic stability problems:

(11) In adaptively controlling a stable plant serious stability problems arise, which have not or only partially been solved so far. This is surprising because any instability in this case can only be generated by the controller itself, i.e. the controller is too "stupid" to recognize his own destabilizing action.

Some of the stability problems in MRAC may be due to its "zero sensitivity" concept:

(12) It is well known from insensitive systems, that (stable) zero sensitivity (by infinite gain) can be accomplished if and only if the plant is minimum phase. The same reasons may be why MRAC stability has been shown so far only for such plants.

(13) Exact model matching is a mathematically tractable concept (having its own limitations and shortcomings) rather than an actual practical need.

Furthermore, it may be physically unfeasible that the plant behaves exactly as a fixed model over a wide range of operating conditions. The latter will be substantiated by an insensitive control example in the subsequent section.

II. SOME ADVANCES IN ADAPTIVE CONTROL

a) Insensitive control for very large parameter ranges

The physical background and the modified sensitivity concept are best introduced by means of an

Example: pitch rate control of a McDonell Douglas F-4C aircraft (longitudinal motion stability augmentation system).

Figure 3 (left curves) show the uncontrolled response of the pitch rate q due to a step elevator deflection for five extremal flight situations. It is obvious, that it would be physically unfeasible to make the response in the

low speed landing approach (Curve 1) as fast as it is possible in a high speed situation (Curves 2, 4). Therefore, maintaining the same model response $q_M(t)$ for all flight conditions cannot be the design goal neither for an insensitive nor an adaptive control design.

Instead, we use the following

<u>goal for insensitive design</u>: $\boxed{\begin{array}{l} q(t) \cong q_M(\alpha_i\ t) \\ \alpha_i \text{ chosen individually for each} \\ \text{flight condition } i \end{array}}$

i.e. we require the response to the basically the same apart from the time scales α_i. Adequate choices of α_i allow the aircraft to behave more slowly in the low speed situation and faster in a high speed one. This is in accordance both with the plant physics and the usual flight control specifications.

Using this sensitivity concept let the transfer behaviour of the controlled plant be $\{u_c \to q_i\}$, $\{u_c \to q_j\}$ for flight conditions i and j respectively. Then, assuming linear feedback the transition from case i to j results in a change of the plant input u by $\Delta u_{ij} = k(\Delta q_{ij})$.

Zero sensitivity now would imply

$$\Delta u_{ij} \neq 0, \quad \Delta q_{ij} = q_{M,i} - q_{M,j} \neq 0 \Rightarrow \text{"finite loop gain"} ,$$

i.e. if zero sensitivity in the above sense could in fact be accomplished by a linear control structure, then this <u>does not require infinite feedback gains</u>, different from the usual comparison sensitivity design discussed in Section 1.

The results of a detail design are depicted in Figs. 3-6. Figure 3 shows the transfer behaviour being well controlled for all flight conditions. Figure 4 shows that the same is true when a step disturbance is added to the system input.

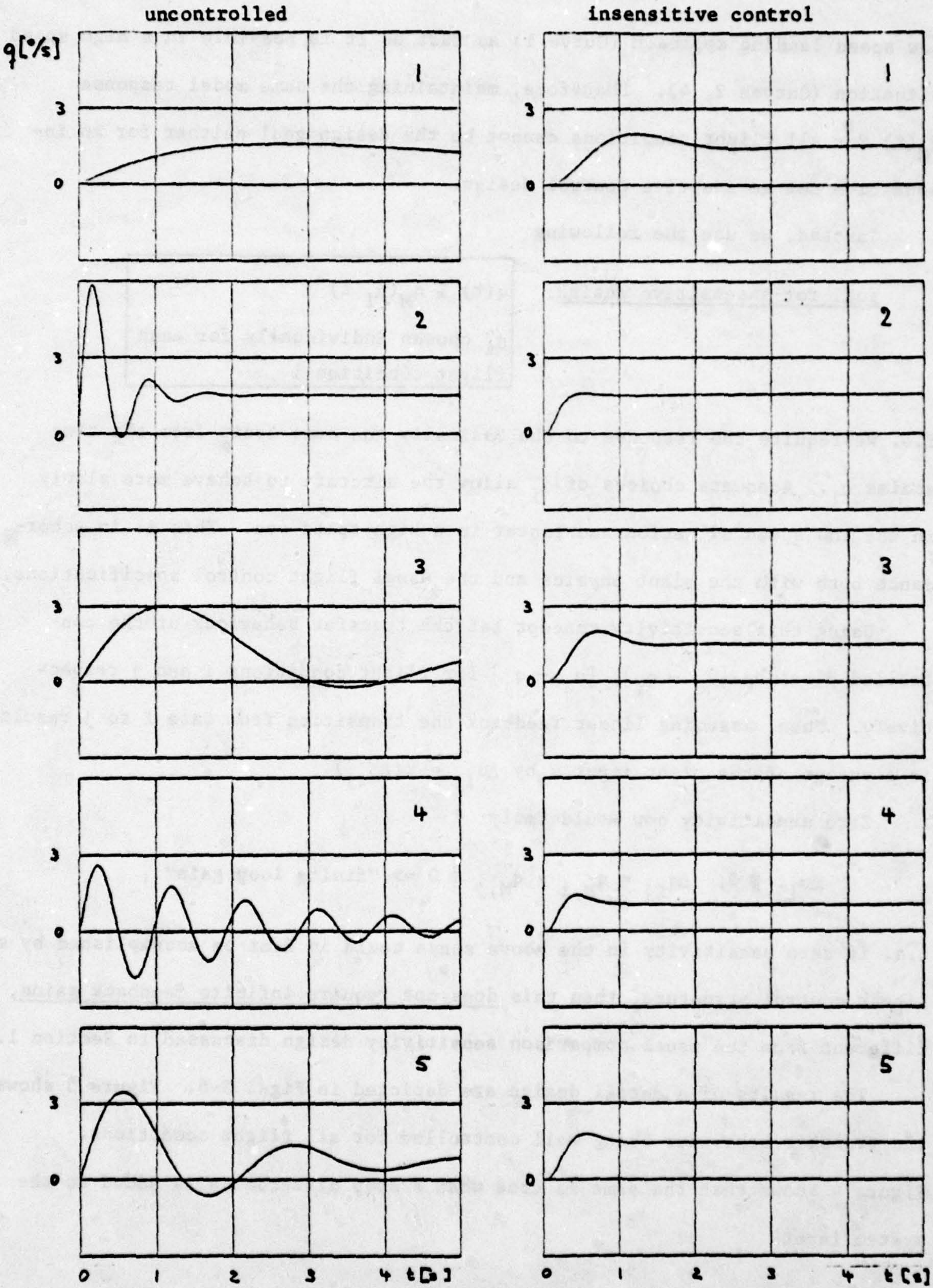

Fig. 3. Response to a step command. F-4 (Phantom) aircraft at 5 extremal flight conditions (altitude 0 ... 40 000 ft, Mach number .2 ... 2.2).

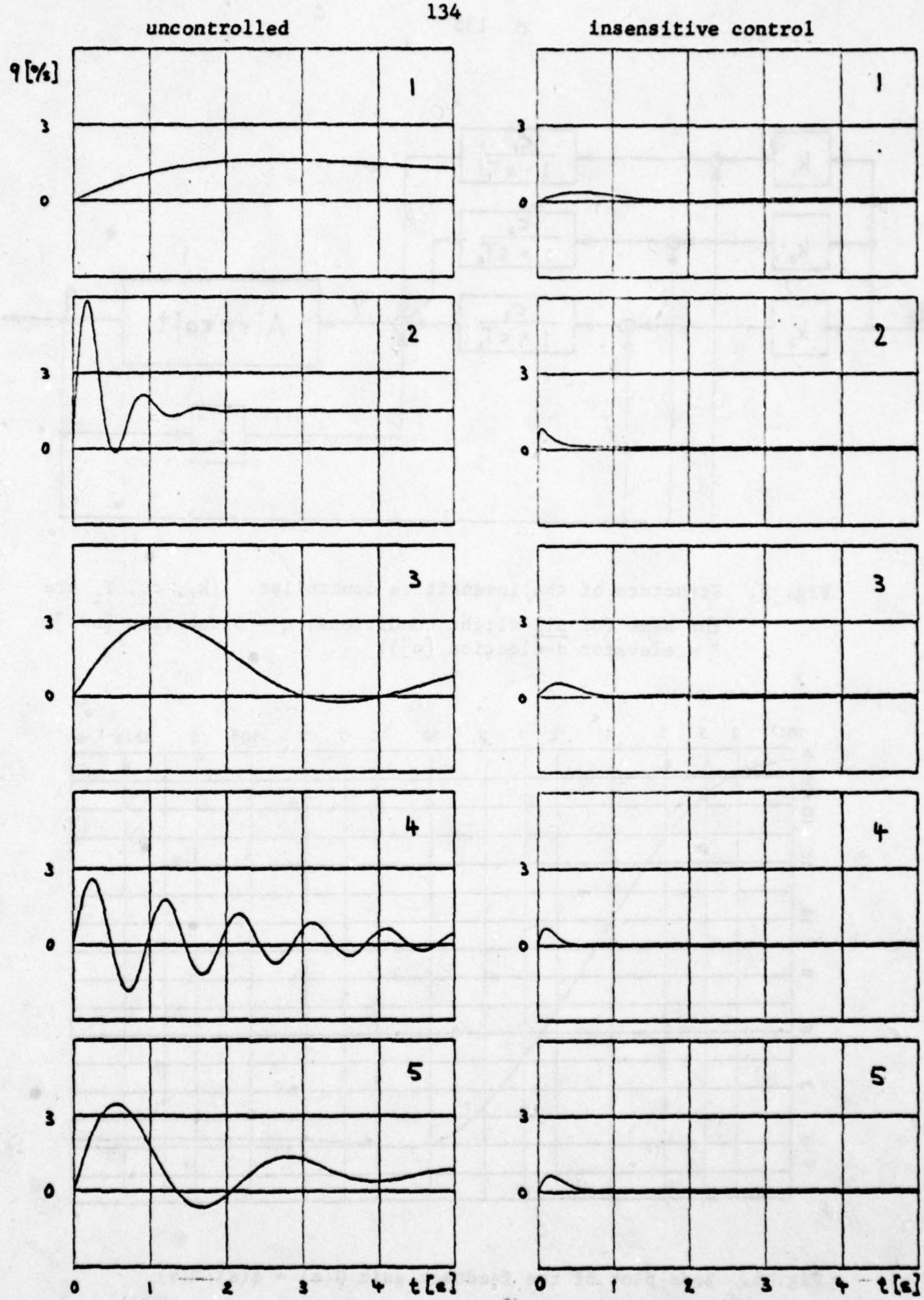

Fig. 4. Response to a step disturbance. F-4 (Phantom) aircraft at 5 extremal flight conditions (altitude 0 ... 40 000 ft, Mach number .2 ... 2.2).

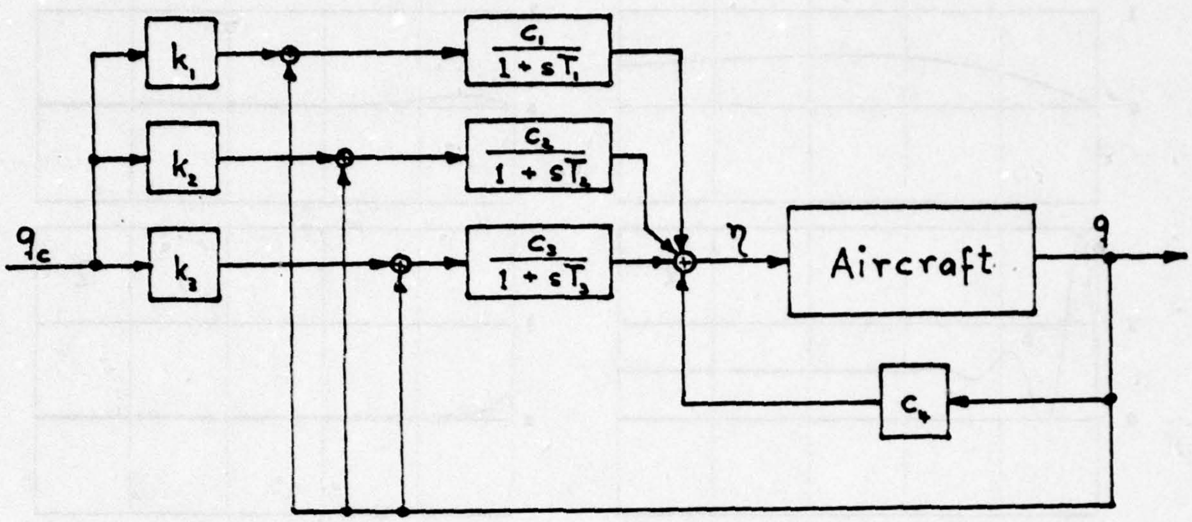

Fig. 5. Structure of the insensitive controller. (k_i, c_i, T_i are the same for <u>all</u> flight conditions; q = pitch rate [o/s], η = elevator deflection [o]).

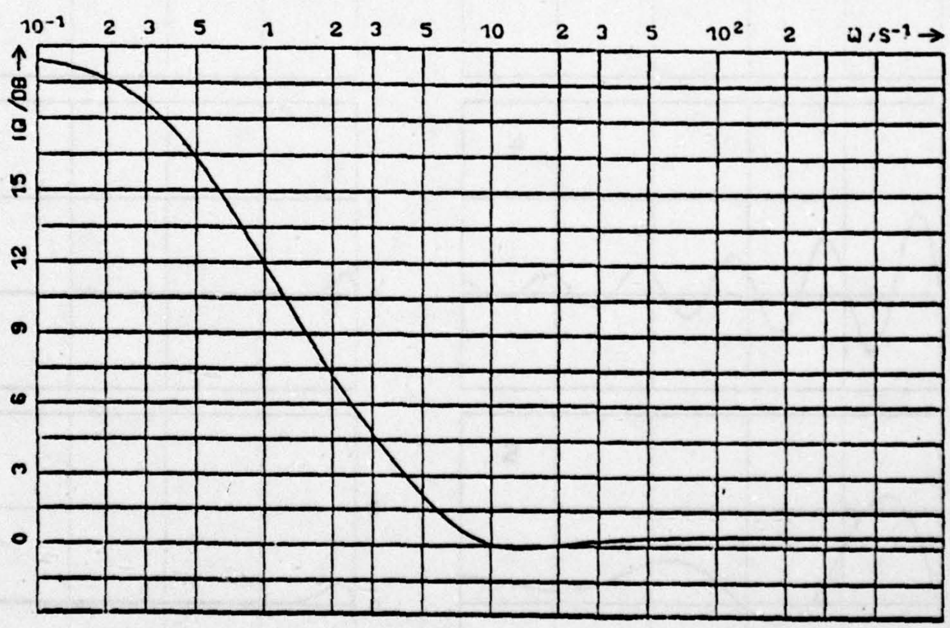

Fig. 6. Bode plot of the feedback gain $G(s) = q(s)/\eta(s)$ (q[o/s], η[o]; $G(j\omega)$ has no phase advance!).

It is important to note, that the controller structure, which is shown in Fig. 5, is extremely simple and does not contain any of the above models. The latter only serve as a means for determining suitable controller coefficients. Finally, Fig. 6 shows, that only low feedback gains are involved. Further design details are contained in [1].

In conclusion, large parameter ranges can well be covered by insensitive control. Furthermore, the physics of the plant must be taken into account also in MRAC (for example, by adapting the model) if large parameter ranges are to be covered. This may require a certain amount of a priori knowledge of the plant even for adaptive systems.

b) <u>An adaptive observer satisfying a separation property</u>

Although stable adaptive observers for unknown linear systems have been available since 1972, no results are known so far about the stability of a state feedback control system, where an adaptive state estimate instead of the true state is fed back. Such a control structure is shown in Fig. 7.

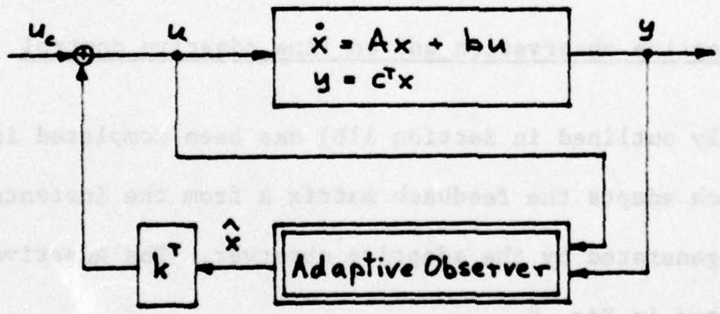

Fig. 7

Closed loop system with an adaptive observer.

The main reasons that the (nonlinear) closed loop stability problem has not been solved are:

(14) The so far available adaptive observers <u>required uniform boundedness</u> of u, y to prove convergence of $\hat{x} \to x$,

(15) but <u>to prove stability of the closed control loop uniform boundedness of u, y cannot be assumed but must be proven</u>.

In [2] a new adaptive observer has been suggested which removes the uniform boundedness assumption and obtains convergence of $\hat{x} \to x$ even in case of u, y increasing unboundly as $t \to \infty$. This is essentially achieved by

(16) <u>additional observation of</u> $\exp(F\,t)[\hat{x}_o - x_o]$ where F contains the observer dynamics

(17) <u>additional control of the adaptive gains</u> (and hence the speed of the adpative process) according to the signal level.

For this adaptive observer the following is shown in [2].

<u>Theorem</u>: If $(A + bk^T)$ is a stability matrix and the adaptive observer is designed to be asymptotically stable, then the control structure of Fig. 7 is also asymptotically stable.

In particular no assumptions on the system parameters, the observer eigenvalues and the speed of the adaptation are made.

A preliminary copy of [2] is available from the author.

c) <u>Adaptive control via adaptive observation and on line adaptive control law synthesis</u>

The work of [2], briefly outlined in Section 11b) has been completed in [3] by an adaptive law, which adapts the feedback matrix k from the instantaneous system parameter estimates generated by the adaptive observer. The adaptive control structure is indicated in Fig. 8.

In [3] a strategy for the adaptive control law synthesis is given, which is shown to result in global asymptotical stability of the control structure of Fig. 8. As an example, the so far unsolved adaptive pole placement problem has been solved in [3].

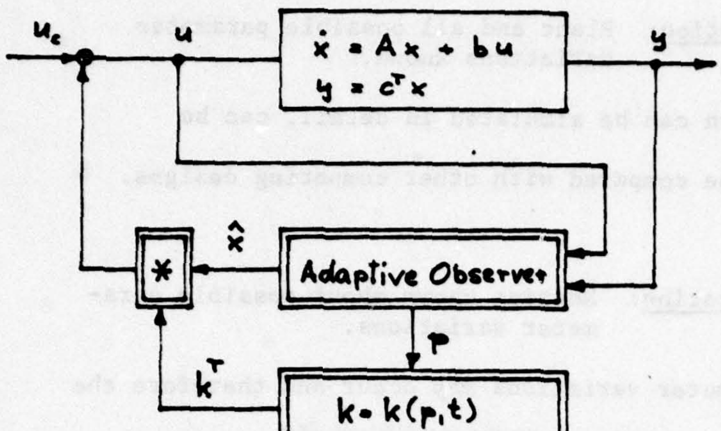

Fig. 8

Adaptive control via adaptive observation and adaptive control law synthesis

These results provide substantial theoretical justification for various practical applications, where similar control structures have been used successfully.

III. ON FUTURE RESEARCH

Adaptive control approaches may be classified into those, who make use of a priori knowledge of the possible plant parameter variations and those, who do not.

Whenever large parameter variations occur in practice they will be well detectable and their main effects will be identified. So, very often a priori knowledge is available in practice. Note that such a knowledge is also necessary whenever the control system is to be simulated on a computer prior to implementing the adaptive controller in practice. The latter should be the rule rather than the exception.

The insensitive control example of section 11a) shows how necessary this knowledge may be, in order that physically unfeasible design goals are avoided. However, most of the adaptive schemes do not make use of such a knowledge and much research should be made how to incorporate it into the design. As a consequence, we have the

<u>first prototype design situation</u>: Plant and all possible parameter variations known.

In this situation a control design can be simulated in detail, can be optimized and in particular can be compared with other competing designs.

Accordingly we have as the

<u>second prototype design situation</u>: Nothing known about possible parameter variations.

In this situation arbitrary parameter variations may occur and therefore the adaptive law must be as skilled as a qualified design engineer firstly to maintain physical feasibility of the control action and secondly to guarantee good regulation accuracy. It is obvious, that this case is much more difficult theoretically and much more risky in its practical implementation at present.

Much more will be necessary to be known about the stability properties of adaptive systems, in particular when unknown, deterministic disturbances act upon the system. Here *deterministic* stability results would be of substantial value, since they also may be a precursor to general results on the disturbance regulation accuracy. Without a well established theoretical stability background adaptive control might be faced with a certain reservation from potential applicants. Also the failure probability would be difficult to analyze.

Finally, we have seen that insensitive control can cover large parameter variations. Therefore it competes with adaptive control, at least as far as the first prototype design situation is concerned. Further developments of such insensitive designs and analysis of the feasible parameter ranges would be of much value. One could also develop insensitive controllers with nonlinear structures, i.e. combine insensitive and adaptive control in such a way that suitable nonlinear controller structures are chosen using adaptive ideas, whereas the design of the free parameters in such structures is based on sensitivity concepts.

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ADAPTIVE CONTROL SYSTEMS, CLASSIFICATION, PROBLEMS, AND SUGGESTIONS*

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1. BACKGROUND

Implementation of control systems using digital logic is of considerable interest because of the present capabilities for designing small, lightweight and inexpensive minicomputers and microcomputers. A feature of digital logic which makes it especially advantageous is the capability for implementation of complex control systems which incorporate high order nonlinearities and multiple loop operations. One such complex control structure is an adaptive system which is capable of on-line adjustment of the control parameters in response to changing plant characteristics.

Two distinct types of adaptive control logic can be considered:

- Explicit adaptive controllers in which on-line estimates of the plant parameters are used for gain adjustment.
- Implicit adaptive controllers in which some measure of the error between actual and desired states is used for gain adjustment, i.e., no explicit parameter identification is used.

Previous studies have indicated the advantages of explicit adaptation if gain magnitudes are constrained and if large parameter variations are to be expected. However, because of the need to incorporate an on-line identifier in such systems, the following issues arise:

- Storage and timing problems in the implementation of on-line identification.

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- How to determine which parameters need to be identified.
- Simultaneous estimation of both states and parameters in a noisy environment.
- Inability to guarantee stability of the controlled system in the presence of imperfect estimates.
- Inability to determine convergence of the identifier when the control signal itself depends upon the identified parameters.

One procedure for alleviating these problems is to use an implicit adaptation algorithm which takes into account system stability requirements. However, such a design requires a means for rapidly assessing performance, which in turn can be used for control gain adjustment. Many investigators have made use of comparisons between actual plant state trajectories and those from a model chosen to reflect desirable operation specifications. Some measure of the error between the plant and model state vector is then directly used for control gain adjustment.

In order to design a controller which does not need explicit parameter estimates, procedures based upon Lyapunov stability and hyperstability principles have been applied to the equations defining the error between plant and model.

However, in order to prove asymptotic stability these designs required the satisfaction of certain structural or matching conditions between plant and model, sometimes known as the conditions for perfect model following (PMF). These conditions are in general not satisfied by most practical plant and model systems. If, for example, the model is chosen to reflect desirable decoupling characteristics not inherent to the process, then the PMF conditions most likely will not be satisfied. Also, if the model is constant and the process is time varying, then it is very likely that the PMF condition will, at most,

be valid only for short intervals of time (e.g., at certain nominal conditions).

To date, limited results have been developed for designing implicit adaptive controllers in the absence of the PMF constraints. At best, it has been shown that a controller can be designed for a multivariable continuous linear system which stabilizes (in the sense of boundedness) the error between the plant and model state.

2. SUGGESTED AREAS OF RESEARCH

2.1 Explicit Adaptive Control System Design

Because explicit adaptive systems can potentially encompass a broad range of performance indices containing penalties on the control gain, it is suggested that research and developmental efforts be geared towards the design of such systems for future computer systems which might alleviate the existing storage and timing problems. In particular, the following activities are recommended:

- Consider the use of parallel architecture for rapid on-line control and estimation.
- Consider the use of indices other than quadratics.
- Consider the design of adaptive controllers for nonlinear systems.
- Implement adaptive control on a physical process.

2.2 Implicit Adaptive Control System Design

Because of the attractiveness of an implicit adaptive control design, it is suggested that research be conducted to increase the applicability of the concept. In particular, the following activities are recommended:

- Design implicit adaptive controller suitable for digital multivariable systems.
- Consider an adaptive controller for output following.

- Consider the effects of noisy state and/or output measurements.
- Design implicit adaptive algorithms for nonlinear systems.
- Demonstrate implicit adaptive control of a physical system.

3. CONCLUSION

In conclusion, it is conjectured that many of the problems previously encountered in designing explicit adaptive controllers will be accommodated using computers employing very large scale integration. However, although implicit adaptive controllers are very attractive because of their relatively simple structure, more analysis is required to extend the range of applicability.

COMMENTS ON ADAPTIVE CONTROL

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Background

During the late 1950's and early 1960's, a lot of effort was put into Adaptive Control research, both theoretical and experimental research. Some of the best known systems were developed by MIT, Honeywell and Autonetics, and several adaptive systems were actually installed and flight tested. The best known flight program was the Honeywell system in the X-15 aircraft. Less well known applications include the F-111 and certain missiles.

The general idea was that an adaptive system should maintain constant dynamic behavior for an aircraft over an extremely wide flight range during which the parameters of the equations of motion were unknown and varying "rapidly." As it turned out, however, adaptive control was overpromoted and oversold. With one possible exception, these systems did not work as advertised.

A successful application required, generally, more knowledge of the dynamics of the airframe than was previously required, not less knowledge, and such things as nonlinearities, turbulence and structural dynamics made successful operation difficult, if not impossible. It was also discovered at that time that invariant dynamic behavior for an airplane over its entire flight range was neither necessary nor particularly desirable.

As a result, adaptive control fell into disfavor in the flight community and to this day a residual bias still exists.

Recent Activity

The recent resurgence of Adaptive Control research is not necessarily directed, in my opinion, towards the solutions of those problems that caused the downfall of the original adaptive control activity and the same pattern of decline can be repeated if we are not careful. In addition to not addressing the old problem, consider some of the new problems associated with modern control theory:

1. Quadratic performance indices for control criteria specification.

   No one has been able to completely successfully relate performance indices to flying qualities. Flying qualities specifications are given in terms of poles and zeros of particular transfer functions of an airplane. A relationship exists between the closed-loop poles and the weighting parameters of a performance index -- but no relationship has yet been found between closed-loop transfer function zeros and performance indices. Non-minimum phase closed-loop transfer functions, often generated by optimal control, are generally to be avoided. How do I choose a performance index that will guarantee minimum phase closed-loop transfer functions?

   -- Failure tolerance. How do I choose a performance index constrained such that stability is still guaranteed if a sensor failure occurs? Will there be time to switch to different control configurations? Should observers be used to replace the lost actual measurement or just output feedback?

2. Identification

   On-line identification is not yet a household word. Our experience with off-line identification has been that accuracy is a maybe kind

of thing, very much dependent upon the particular airframe, the quality and complement of instrumentation on the aircraft, the feedback control law, the model form and the input design. The particular identification algorithm used is less a factor than the realities of the plant itself. The same flight data can be analyzed for weeks, adjusting such things as noise covariances and instrument biases, and increasingly more accurate results are obtainable. Occasionally, after the first successful identification has been laboriously obtained, subsequent identifications can be performed in one pass, but only within a limited flight range.

Consider some of the requirements for accurate identification.

1. The excitation to the system must be such to render each of the parameters identifiable. Can successful identification be accomplished using normal pilot inputs or actual turbulence excitation? Only a flight test program can answer this question.
2. State estimation should be accomplished independently of aerodynamic parameter identification, particularly if the aerodynamics are nonlinear. This is to verify the consistency of the instrumentation complement. Instrumentation scale factors and bias errors change with such things as angle of attack; gyros drift, the c.g. of the aircraft changes, etc.
3. The model form changes. An aircraft that <u>should</u> be considered 4th order at slow speed <u>may</u> be considered a 2nd order system at intermediate speeds, but <u>must</u> be treated as a nonlinear system at transonic Mach numbers or at high angle of attack. Do we do on-line hypothesis testing to determine the correct model form?

4. We have no idea how accurate the parameter estimates must be to either guarantee stability or to insure good flying qualities. How accurate is accurate enough?

Very important will be the cycle time for a complete computation; estimation and identification or hypothesis testing and control law adjustment. For most manned aircraft, our experience has been that a transport lag between the time that a pilot puts a command into the airplane until the time that the airplane starts responding to that command must not exceed 40-60 milliseconds and is a function of the vehicle stability. In generaly, any delay is undesirable.

Need for Experimental Research

Many of the questionable aspects of adaptive control, particularly with respect to manned aircraft, can only be answered by an experimental research effort that parallels the theoretical work. Adaptive control suffers from a credibility gap among many users that may be closed by a carefully calculated series of flight experiments designed to verify, one step at a time, the developments of recent years. A common basis for evaluation of competing ideas is necessary. Flight experimentation provides exposure to the flight community and is the first step towards acceptance. This step is necessary -- without it adaptive control research will likely fade away as it had in the past. Adaptive control is too promising to risk the fate of being relegated to the status of a theoretical tinker-toy.

I would caution against the desire for an exotic application. Although some considered the F-8 airplane application to be less successful than it might have been because the F-8 had acceptable flying qualities and, therefore, did not require an adaptive system, in fact the F-8 seemed a good "test bed" and the argument is not valid. The flying qualities of any airplane can be

improved with feedback. This objective should have been an acceptable one within the requirement for safety of flight. The purpose was not to replace the original F-8 flight control system, but to show that an adaptive system can work.

Any airplane that has an independent controller for each degree of freedom of motion and measures a full state vector should be an acceptable "test bed." If it is required that the subject aircraft have poor dynamics, this can be done to any aircraft either with "hard-wired" feedback, by adding lead weight to the tail section, or by other means.

There seems little doubt that a select series of flight test experiments at this time will do more to not only define the future directions for adaptive control, but also to create a climate of acceptance by the flight community.

ADAPTIVE CONTROL OF NON-MINIMUM PHASE PLANTS: A REAL PROBLEM*

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As currently developed, self-tuning [1] and model reference adaptive controllers [2] rely on eventual plant numerator cancellation in order to achieve their established control objectives. This characteristic has limited stability studies and applications of adaptive control to plants with stable inverses, typically denoted as non-minimum phase plants. (As an aside, note that the non-minimum phase designation arising from frequency response magnitude/phase relationships [3] actually requires all plant singularities, both poles and zeros, to be stable. Common usage has narrowed this term to description of zero locations.) The number of physical plants with non-minimum phase behavior, in aerospace and industrial process applications among others, makes this narrow focus a real problem.

Currently four approaches exist which enjoy limited success in adaptive control of non-minimum phase plants: (i) simultaneous identification and control (SIC) (also termed indirect adaptive control), (ii) input matching (IM), (iii) adjustable model reference adaptive control (AMRAC), and (iv) delayed model control (DMC).

SIC: Convergence justification of all SIC schemes relies on sufficient identifier excitation for exact plant parameter estimation. This restriction also requires, in general, plant model minimality for unique parameterization. Despite these two restrictions such indirect SIC schemes currently enjoy the broadest possibility of control objectives restricted only by what controller

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design can be achieved on-line in real time from plant parameter estimates. For example, a SIC scheme requiring on-line factorization has been proposed [4]. Recently a significant breakthrough [5] in the stability analysis of the separation basis of adaptive observation and state feedback has advanced the possibility of SIC provability. Extension of these separation results to SIC has been proposed [6].

IM: Currently IM [7] rests on single-stage quadratic cost function minimization (as do equivalent techniques arising from similar but distinct concepts [8] [9]) permitting control of some but not all non-minimum phase plants [10]. The global stability of adaptive IM is currently unproven due to the reliance of three recently developed discrete-time stability proofs [11][12] [13] on plant inverse stability. The extension of IM to a longer horizon quadratic cost function appears to require further plant numerator parameter knowledge than the leading parameter knowledge currently deemed necessary. Ongoing studies, e.g. [14], of the full flexibility of single-stage cost functions may improve the stabilizability afforded via judicious modifications.

AMRAC: The concept of AMRAC is to limit the adaptive controller cancellations of plant singularities to stable values and adjust the reference model to incorporate the unalterable unstable values. All adaptions are to be based on the error between the model and plant-controller outputs. The logical AMRAC approach has apparently borne fruit in pole cancellation and replacement [15] and pole-shifting via state feedback [16] applications. The two current major drawbacks of extension appear to be: (i) restriction to stable plants and (ii) possible unstable cancellations in most general controller structures.

DMC: Recognition of the effective delay inherent in non-minimum phase responses prompted the inclusion of a bulk delay in the reference model to be

followed in order not to try and undo the non-minimum phase effect at tremendous control input cost. One application of this approach has been suggested [17] by cascading a MA controller and a delayed AR model of a non-minimum phase ARMA plant. The delayed AR model allows compensation of the non-minimum phase zeros of the plant by cancelling after time-shifting the plant's decaying two-sides AR model impulse response. Such a scheme does not presently appear to lend itself to the inclusion of feedback control elements.

Further evaluation of these concepts or development of new ones to address the non-minimum phase plant control problem represents one of the most immediate tasks of adaptive control research.

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ON ADAPTIVE CONTROL

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In a round table session on adaptive control held recently at a national conference I observed that it is not possible on the basis of an examination of a control system to determine whether or not it is adaptive. This is a logical impossibility, nor merely a practical problem. In order to determine whether a system is adaptive it is necessary to know not only the procedure used by the designer, but also the terminology he used.

The very use of feedback in the design of a control system has the effect of reducing the sensitivity of the system to external disturbances and to changes in characteristics of the process. The behavior of the system tends to be invariant to changes in itself or in the environment and hence it is legitimate to call such a system "adaptive" in the customary usage of that word.

In the context of control theory, however, the usage of the word "adaptive" is usually defined in such a way as to exclude control system designs in which the "state variables" or related dynamic variables are measured, but in which the "parameters" are assumed known. But the distinction between state variables and parameters is a convenience of the control system designer and not necessarily an objective fact. For example, in the process

$$\dot{x} = - px + u \qquad (1)$$

everyone would call the variable x the "state" and the variable p the "parameter." But suppose p is not exactly constant: say,

$$\dot{p} = xv \qquad (2)$$

where v is another input variable (either a control or a disturbance). Now the analyst has two options:

(A) He can design an adaptive control system for (1) by either tracking p or by making the design robust (so that a reasonable range of variation of p can be accommodated).

(B) He can design a non-adaptive control system for the process consisting of (1) and (2) together.

An individual who examines the resulting system should have no trouble gauging how well it performs but would have *no* way of knowing whether the system is adaptive.

The mechanism of feedback provided all the adaptation needed in technology until very recently (i.e., until after World War II) when availability of sophisticated electronics (analog and digital computers) pointed to the possibility of improving system performance at the expense of increasing hardware complexity.

The earliest requirement for adaptive control arose in the technology of military aircraft flight control where maneuverability requirements of aircraft can be achieved only at the expense of *open-loop dynamics* that verge on instability. A feedback control system can improve dynamic performance but is sensitive to vehicle parameters. A very reasonable engineering approach is to design the control system parameters to be known functions of the vehicle parameters, and to track the latter in flight, thereby keeping the control system "in tune" with the actual aircraft parameters. Many of the desired parameters (i.e., "stability derivatives") are reasonably systematic functions of a small number of fundamental variables, primarily dynamic pressure $q = \rho v^2/2$, so a good deal of adaptivity can be provided by simply measuring dynamic pressure and scheduling the control system gains accordingly. The relative simplicity

of this technique disqualifies it, in the eyes of some, from being classified as adaptive.

At the other extreme is the "universal bionic controller" (UBIC) a box with hundreds of input and output terminals and with a very fast, large-memory computer inside it. No a priori knowledge of the process is assumed, signals from everything that can be measured are connected to the input terminals, and signals from the output terminals are brought to everything that can be moved. At $t = 0$, a start button is pushed. In a short time the controller has learned the process dynamics, the parameter values, and has determined the control parameters of the optimum control law. It keeps cycling through this sequence so that parameter changes or even structural changes in the process are discovered and the control law is recomputed almost instantaneously. Perhaps such a box can be built, but those of us who are around to see it work will be engaged in other intellectual pursuits.

The problems that merit current attention are somewhere between those that can be solved by gain scheduling and those that require a UBIC. Dynamics of processes in this category, I would think, are those for which there is a general concensus about which variables are state variables and which are parameters. And the form of mathematical model that represents the process is reasonably well understood. Moreover, it should be generally agreed that the application justifies adaptive control because the process is either difficult to control by less sophisticated methods, as is the case with high-performance aircraft, or because the economic benefits of slight improvements in performance justify the outlay for an adaptive control system.

ON STOCHASTIC ADAPTIVE CONTROL

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When we want to design control laws for stochastic systems with unknown parameters, disturbance inputs and measurement noise, two roles of the control input have to be considered. The first one is to excite the system to produce outputs which are associated with desirable system performance, and another is to produce outputs from which good estimates of the unknown parameters can be obtained. The kind of controllers that takes into account these two roles is called a dual controller. To find the optimal controller a closed loop controller has to be designed because it anticipates that future measurements will be taken and it achieves a compromise between the control and the estimation objectives. Due to the fact that the solution to the optimal control problem is computationally impossible, various suboptimal algorithms have been developed that have the dual property. Some use explicitly in the cost function the conflicting characteristic of the control [1-4] and others are approximations of the dynamic programming equation and lead to a cost function where the conflicting characteristic of the control is evident [5]. In the first classification we include the safer control which not only takes into account the conflicting characteristic of the control in the cost function but also, through the design of an output sensitivity weighting matrix, redistributes the estimation effort according to the accuracy required to achieve a given control objective. In this method we find a control and a sensitivity weighting matrix that will tend to reduce the sensitivity of the output with respect to the unknown parameters, while tending to achieve output regulation.

The problem with the dual control algorithms developed up to now is computational complexity. A desirable direction for future research is to seek computational simpler dual adaptive control algorithms and to develop control laws that enhance estimation for a class of nonlinear stochastic control problems.

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A MINIMAX APPROACH TO THE DUAL CONTROL PROBLEM

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Consider the following statement of a dual control problem. It is desired to design a closed loop control for the system

$$x_{k+1} = A_k(\theta)x_k + B_k(\theta)v_k; \quad x_0 = x(0); \quad k = 1,2,\ldots,N \tag{1}$$

$$y_k = C_k(\theta)x_k + D_k(\theta)w_k \tag{2}$$

in such a way as to minimize the maximum of the incremental quadratic performance index

$$R(u,\theta) = J(u,\theta) - J(u_\theta^*,\theta) \quad \text{over all } \theta \in \Theta \tag{3}$$

where

θ is a vector of constant but unknown parameters. It is further assumed that θ is an element of a known compact subset Θ of $\mathbb{R}^P$. Θ may be convex and no prior probabilistic information on θ is assumed.

x and y are scalars.

x(0) is a scalar Gaussian random variable whose mean and variance are known functions of θ.

w_K is a scalar-white Gaussian noise process whose mean and covariance are also known functions of θ.

$\underline{v} \triangleq [v_1,v_2,\ldots,v_N]'$ is the vector of controls over the horizon 0 to N resulting from the feedback control law u(Y).

Y is the measurement set $[y_1,y_2,\ldots,y_N]'$.

u_θ^* is the control law which minimizes $J(u,\theta)$ for known θ.

$$J(u,\theta) \triangleq \frac{1}{2} E\{Q_1 x_N^2 + \sum_{i=1}^{N} [Q_2^{(i)} x_i^2 + Q_3^{(i)} v_i^2] | \theta\}$$

A scalar structure is posed to simplify the mathematical exposition. Extension to the vector case presents no essential difficulties.

It is worthwhile to carefully consider the implications of this problem statement. In the first place, a feedback control of the form

$$\underline{v} = u(Y) \tag{4}$$

is desired. Secondly, no plant noise is allowed. This is a useful formulation in many control problems (e.g. control of spacecraft suffering from launch uncertainties). While presently essential, it is expected that the latter restriction will yield with further study. Thirdly, we assume that the parameter vector θ is known only to lie in a closed and bounded, possibly convex subset of $\mathbb{R}^P$ and that no other prior information is available. In particular, Bayes solutions are impossible and no intelligent adversary is assumed to control θ. Finally, the choice of performance index is crucial. As shall be demonstrated below, it is convenient to attempt to minimize the maximum performance difference between a given control u and the optimal control u_θ^* which could be used if θ were known. $R(u,\theta)$ is computationally attractive and it permits formulation of the problem as a minimax problem without requiring choice of θ by an intelligent adversary. Consider the qualitative description of Fig. 1 for a scalar $\theta \triangleq [\theta_1, \theta_2]$. The optimal controller u^* satisfying (3) would attempt to provide a performance $J(u^*,\theta)$ which would match $J(u_\theta^*,\theta)$ as closely as possible over the entire Θ. It would do so by paying a slightly higher price than $\bar{u}$ at $\theta = \theta_2$ in return for a reduction in the cost for $\tilde{u}$ near $\theta = \theta_1$. It very probably would not equal $J(u_\theta^*,\theta)$ for any $\theta \in \Theta$. Using a minimax criterion on the incremental quadratic

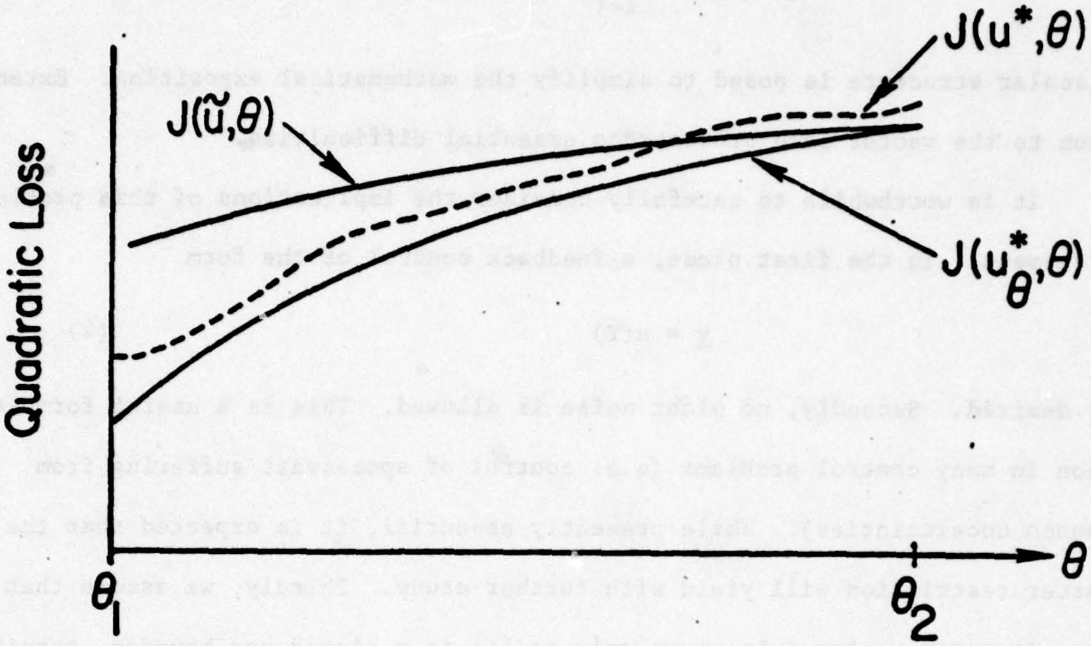

Figure 1. A qualitative comparison of the performance of various controllers:

u^*_θ minimizes $J(u,\theta)$ for a fixed θ

$\tilde{u}$ is minimax for $J(u,\theta)$ over $\theta \in \Theta = [\theta_1,\theta_2]$

u^* is minimax for $R(u,\theta)$ over $\theta \in \Theta$.

loss function $R(u,\theta)$ effectively selects that controller which performs as close as possible over all θ to the best one could do with complete knowledge of θ - a very desirable performance characteristic. Fortuitiously, it is also computationally tractable. A similar incremental loss function has been used for some time in the design of minimum sensitivity controls for deterministic systems [1].

II. The Optimal Solution

For several reasons, it is convenient to view (1)-(3) as a game between the statistician (who chooses u(Y)) and nature (who chooses θ). As discussed above, (3) permits the use of a minimax criterion without its usual pessimism.

Secondly, a great deal is known about the use of game theory in the determination of minimax solutions to problems like (1)-(3).

A game is completely specified by a pair of strategy spaces from which each player extracts his action and a loss function which determines the payoff for each possible pair of actions. Assuming the loss function (3), a minimax statistician's solution to the game would satisfy:

$$\min_{u} \max_{\theta} R(u,\theta) \tag{5}$$

Unfortunately, this optimization is difficult since the minimization must be performed over a function space (the statistician's strategy space). If $\min_{u} \max_{\theta} R(u,\theta) = \max_{\theta} \min_{u} R(u,\theta)$, and if $\min_{u} R(u,\theta)$ could be easily determined for a fixed θ, then the optimization (5) becomes a much simpler parametric one. The first condition holds if the game has a value. The second holds if the *minimax solution to the game is equal* to the Bayes solution under some computable prior. The latter, if it exists, is called the least favorable prior for the game. Finally, of course, the minimax solutions must exist. Since a well behaved game has all of these properties, the solution to (1)-(3) will be achieved if it can be cast in the proper framework.

The first difficulty is to determine the strategy spaces for both players. Θ is a reasonable choice for nature. A first choice for the statistician's space might be $\mathcal{U} \triangleq \{u\colon \operatorname{tr} E\{\underline{v}^T\underline{v}|\theta\} < \infty \;\forall\; \theta \in \Theta\}$. Since neither Θ nor $\mathcal{U}$ are finite spaces, the game $(\Theta,\mathcal{U},R)$ is difficult to solve. Also, the use of pure strategies such as u and θ do not always guarantee the desired properties. One is therefore at least initially forced to consider randomized strategy spaces.

In the present context, a randomized strategy for the statistician would be the choice of a probability distribution on the elements of the given

function space rather than a particular function u(Y). Similarly, a randomized strategy for nature would be the choice of a distribution of $\theta \subset \Theta$ rather than a choice of a specific value of θ. Fortunately, randomizations on the statistician's space will not be necessary.

In the sequel, we shall define a subset, $\mathcal{U}_M$, of $\mathcal{U}$ which is weakly compact and demonstrate that $R(u,\theta)$ is weakly lower semi-continuous for all $u \in \mathcal{U}_M$. This choice of both $\mathcal{U}_M$ and the weak topology are the key to insuring that the resulting game $(\Theta,\mathcal{U}_M,R)$ can be used to solve (1)-(3). After introducing some notation, this result will be formalized in a Theorem.

Let:

$\mathcal{Y} \triangleq$ space of all possible observations sets $Y = \{y_i: i = 1,2,\ldots,N\}$.

$\mathcal{U} \triangleq$ space of all functions $u: \mathcal{Y} \to \mathbb{R}^N$ such that

$$\|u(Y)\|_{\mathcal{U}} \triangleq \operatorname{tr} E\{\underline{v}^T\underline{v}|\theta\} < \infty \;\forall \in \Theta .$$

$$\mathcal{U}_M \triangleq \{u: \mathcal{Y} \to \mathbb{R}^N \;|\; \|u(Y)\|_{\mathcal{U}} \leq M < \infty \;\forall\; \theta \in \Theta\}.$$

$$\bar{J}(u,\theta) \triangleq J(u,\theta)\Big|_{x(0)\equiv 0} \text{ a.e.}$$

$$\mathcal{J} \triangleq \{u: \mathcal{Y} \to R^N \;|\; u \in \mathcal{U} \text{ and } J(u,\theta) < \infty \;\forall\; \theta \in \Theta\}.$$

$\Theta \triangleq$ space of allowable values of θ.

$\Theta^* \triangleq$ space of all probability distributions on Θ, i.e.,

$$\{\tau(\theta): \theta \in \Theta, \tau \text{ is a valid probability distribution and } \tau(\Theta) = 1\}.$$

$\mathcal{U}_M^* \triangleq$ space of all probability distributions on $\mathcal{U}_M$, i.e.,

$$\{\gamma(u): u \in \mathcal{U}_M, \gamma \text{ is a valid probability distribution and } \gamma(\mathcal{U}_M) = 1\}.$$

We can therefore define a game $G = (\Theta,\mathcal{U}_M,R)$ for the purpose of finding a minimax control law $u^* \in \mathcal{U}_M$ satisfying

$$\sup_{\tau \in \Theta^*} E_\tau\{R(u^*,\theta)\} \leq \sup_{\tau \in \Theta^*} E_\tau\{R(u,\theta)\} \; \forall \; u \in \mathcal{U}_M \tag{6}$$

where $E_\tau\{\cdot\}$ denotes expectation with respect to the density τ. The minimax u^* and several of its crucial properties are given in the following Theorem which is proved in [2].

Theorem 1:

(i) The non-randomized control laws $\mathcal{U}_M$ form an essentially complete class and therefore randomized control laws need not be considered.

(ii) The game $(\Theta,\mathcal{U}_M,R)$ has a value.

(iii) There exists a least favorable prior $\tau_0 \in \Theta_d^*$ where Θ_d^* is the space of all discrete probability distributions on Θ having a finite number of points of positive support.

(iv) u^* is Bayes with respect to τ_0.

This is indeed a very remarkable result. It demonstrates that the optimal solution to the reformulated dual control problem is Bayes with respect to a finite dimensional prior even though it is both multivariate and convex.

Theorem 1 guarantees the existence of both the minimax controller and the least favorable prior and specifies the structure of the optimal controller thereby converting the original function space optimization problem to one of parameter optimization. In particular, since $(\Theta,\mathcal{U}_M,R)$ has a value, it is sufficient to determine the $u \in \mathcal{U}_M$ which satisfies

$$\sup_{\tau \in \Theta_d^*} \inf_{u \in \mathcal{U}_M} E_\tau\{R(u,\theta)\}$$

which by Theorem 1 is equivalent to finding $\tau_0 \in \Theta_d^*$ for which

$$E_{\tau_0}\{R(u^*,\theta)\} \geq E_{\tau}\{R(u^*,\theta)\} \;\forall\; \tau \in \Theta_d^* \qquad (7)$$

Note that provided the Bayes control is available, the only unknowns in (6) are the K points of positive support of τ_0 and the K-1 probabilities which τ_0 applies to them. The methodology of (6) is robust since it is optimal for compact parameter spaces and for all LQG systems admitting an optimal controller with finite loss for all $\theta \in \Theta$.

Corollary 1: u^* is approximately given by [3]:

$$u^* = \frac{\hat{u}_1 + \sum_{j=2}^{K} \hat{u}_j \Lambda_{1j}}{1 + \sum_{j=2}^{K} \Lambda_{1j}} \quad \text{for some } K < \infty \qquad (8)$$

where

$\hat{u}_i = - P(\theta_i)m_i$, the optimal solution to the LQG problem for $\theta = \theta_i$;

θ_i are the points of positive support of $\tau_0 \in \Theta_d^*$;

$m_i = E\{\underline{x} \triangleq [x_1, x_2, \ldots, x_N]' \,|Y, \theta\}$, the minimum mean square error (MMSE) estimate of $\underline{x}$ given $\theta = \theta_i$;

$P(\theta_i)$ is the optimal controller gain for the LQG problem with $\theta = \theta_i$;

$\Lambda_{1j} = \dfrac{dF(Y|\theta_j)p_j}{dF(Y|\theta_1)p_1}$ is the generalized likelihood ratio;

$dF(Y|\theta)$ is the conditional probability measure of Y given θ;

p_i is the probability mass given $\theta = \theta_i$ by τ_0.

Summary and Conclusions

1) In the design of adaptive controllers, an incremental quadratic loss function is useful in that it permits use of non-pessimistic minimax techniques.

2) It has been demonstrated using Hilbert space and game theoretic techniques that the optimal solution to the dual control problem (1)-(3) is Bayes with respect to a least favorable prior having a finite number of points of positive support. Furthermore, it has the same structure as the standard Bayes controller for $J(u,\theta)$ of (3).

3) The resulting optimal structure lends itself readily to suboptimal solutions.

4) Results contained herein significantly reduce the class of Bayes problems of relevance to optimal dual control problems.

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ON CONTROL RESEARCH

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Comment 1

In addition to the need for continued support of research in the more established areas of stochastic adaptive control, I feel that it is also important that the Air Force support research in the area of decentralized and hierarchical adaptive control--for, from this research could arise important new classes of self organizing control systems.

Comment 2

The control community needs a generic class of problems at different levels of complexity *that potentially*, at least, require algorithms based upon stochastic adaptive control theory. This class of problems should exhibit a diversity of complexity with respect to specified performance measures.

A set of evaluation procedures should be precisely specified. In addition to quantifying conventional measures of performance, these procedures should quantify how matters such as robustness to various parameter sets, operating regimes, nonlinearities, etc., are to be explicitly evaluated. The evaluation procedure should also require a quantitative assessment of the resulting control algorithms relative to their (1) computer hardware and software requirements (in terms of some given generically defined computer system), and (2) their simplicity per se as well as their ease of use by control practitioners other than the author(s) of the algorithms.

Wherever possible, simple benchmark algorithms should be supplied.

The problems should be <u>based</u> upon important practical problems, but should be stated in such a manner that virtually any member in the community of control researchers could address the problem of possibly contributing an algorithm to the "competition." Each possible contributor would then start with essentially the same state of knowledge relative to the problem statement <u>and</u> the quantatively defined evaluation criteria.

These requirements, while admittedly being classificable as "difficult," are well within the capabilities of the control community. The expected benefits of such a program make it well worth the effort required to establish it.

MACROECONOMIC POLICY MODELING AND ADAPTIVE CONTROL

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Abstract

Lucas [6] has recently argued that government policy planning undertaken with traditional macroeconomic policy models is unreliable, since no allowance is made in these models for the reaction of other rational decision-making agents in the economy to changes in government policy. However, the incorporation of symmetrical rationality into macroeconomic policy models leads to a difficult adaptive control problem for government planners requiring the on-line estimation of control and state dependent coefficients. Three alternative approaches to the problem are briefly mentioned.

Consider the following macroeconomic policy model format,[1] typically used ([1], [3], [4]) to describe the policy choice problems facing government planners. The observed motion of an economic system over N periods is described by a difference equation

$$x_1 = \bar{x} \text{ (initial conditions)}, \tag{1a}$$

$$x_{n+1} = f_n(\omega_n, v_n, x_n),\ 1 \leq n \leq N, \tag{1b}$$

[1]Macroeconomics is the study of major economic aggregate variables such as total production (GNP), total employment, the average price level of all goods and services, and the total money supply. Macroeconomic investigations generally focus on two important concerns: The causal relationships among the aggregate variables; and the prediction of the effects on these aggregate variables of alternative government agency policies (control actions). Models designed for the latter purpose are referred to as macroeconomic policy models.

where the nth period <u>system state</u> x_n is an element of R^s, the nth period <u>government control</u> v_n, to be announced at the beginning of period n, is constrained to lie in an <u>admissible control set</u> $V(n,x_n)$ contained in R^c, $\omega_n \in R^r$ is a random vector composed of unknown parameters in equation (1b) together with a residual error term, and the <u>state function</u> $f_n: R^{r+c+s} \to R^s$ is continuous. The value associated with each possible configuration (ω_n, v_n, x_n) for period n is measured by a continuous <u>return function</u> $W_n: R^{r+c+s} \to R$. Finally, the set $\mathcal{L}$ of admissible feedback control laws for system (1) consists of all vectors $\underset{\sim}{v} = (v_1(\cdot),\ldots,v_N(\cdot))$ of measurable functions $v_n: R^s \to R^c$ satisfying $v_n(x) \in V(n,x)$ for each n and x.

The traditional approach to macro policy modeling assumes that each random vector ω_n is an independent drawing from a possibly degenerate probability distribution $(R^r, \mathcal{B}, p_n)$ that is known (estimated) by the planner prior to period 1. The planner in period 1 thus faces an ordinary stochastic control problem: Maximize expected total return

$$E[\sum_{n=1}^{N} W_n(\omega_n, v_n(x_n), x_n)|\bar{x}] \qquad (2)$$

subject to (1) by selection of feedback control law $\underset{\sim}{v} \in \mathcal{L}$.

Recently, however, it has been emphasized by Lucas [6] and others that this traditional approach to macroeconomic policy modeling ignores important game aspects inherent in economic planning. Realistically, the random vectors ω_n in (1b) should be interpreted as functions of the demand and supply decisions of other optimizing agents in the economy who react rationally to changes in government policy and the state of the economy. Specifically, letting I_n denote the nth period information set consisting of past and current control and state realizations $v^n \equiv (v_1,\ldots,v_n)$ and $x^n \equiv (x_1,\ldots,x_n)$, in

addition to the relevant system dynamics (1), the nth period random vector ω_n might reasonably be assumed to be governed by a more general Stackelberg probability distribution of the form $\langle R^r, \mathcal{B}, p(\cdot|I_n)\rangle$.

If the distributions $p(\cdot|I_n)$ are known to the government planner in period 1, then in principle he still faces an ordinary stochastic control problem. However, although it is conceivable that satisfactory control performance might be achieved over periods $\{1,\ldots,N\}$ following the prior off-line estimation of distributions $p_n(\cdot)$ exhibiting only time dependencies (e.g., stable periodicities), it is entirely inconceivable that general distributions of the form $p(\cdot|I_n)$ could be estimated off-line, prior to any control action by government. Rather, the government planner's assignment must now realistically be viewed as a difficult on-line adaptive control problem involving simultaneous learning and control, a task for which traditional macroeconometric estimation techniques are ill-suited.

In Refs. [8-9] an adaptive control method is developed which is applicable to this problem, assuming the existence of (unknown) Markov transition probabilities $p(\cdot|I_n) \equiv p_n(\cdot|v_n,x_n)$ and the observability of the realizations ω_n. The key distinguishing feature of the method is the direct estimation and updating of the relevant dynamic programming optimality equations in each period n without resort to explicit probability distribution specification.

An alternative game-theoretic approach is suggested in [10] and developed in [5] in the context of a C^3 (command, control, and communication) model having the basic format (1) and (2). The vectors ω_n in (1b) are interpreted as the realization for an unknown feedback control law $\underset{\sim}{\omega}$ implemented by an opposing player. A Markov transition probability $p_n(\cdot|x_n)$ is generated for ω_n on the basis of a probabilistic assessment over opposing player preference structures

and an iterative derivation for the (unique) Nash equilibrium strategy pair $(\underset{\sim}{v}, \underset{\sim}{\omega})$ corresponding to each preference specification.

Nevertheless, both of these approaches are open to criticism in <u>macro</u>-economic contexts, the first due to reliance on <u>observable</u> realizations ω_n, and the second due to implicit reliance on the existence of a <u>single</u> (or representative) opposing player.

Macroeconomists have recently begun to explore a third alternative involving prior linear restrictions on the form of x_n, v_n, and ω_n (see [2] and [7]). For illustration, suppose the state equations (1b) take the linear form

$$x_{n+1} = Ax_n + \omega_n + Cv_n, \; n \in \{1,\ldots,N\} , \tag{3}$$

where A and C are known constant coefficient matrices, and suppose government is restricted to linear control laws of the form $v_n(x_n) = G_n x_n$. Finally, letting $E[\cdot|I_n]$ denote expectation with respect to $p(\cdot|I_n)$, suppose ω_n takes the form

$$\omega_n = B_1 E[x_{n+1}|I_n] + B_2 E[x_n|I_{n-1}] + \epsilon_n , \tag{4}$$

where B_1 and B_2 are known coefficient matrices and ϵ_n is a white noise zero-mean process. The interpretation of (3) and (4) is that the behavior ω_n of other agents in the economy enters linearly into the state equation (3) in the form of state expectations which are "rational" in the sense that they are consistent with the true state generating mechamism (3). Substituting for v_n and ω_n in (3), and taking expectations with respect to $E[\cdot|I_n]$, $n \in \{1,\ldots,N\}$, the expectations in (4) can be eliminated by backward recursion. The resulting reduced form state equation takes the form

$$x_{n+1} = Q(G_n)x_n - [I-B_1]^{-1}B_2 Q(G_n)\epsilon_{n-1} + \epsilon_n , \tag{5}$$

where I denotes the identity matrix and

$$Q(G_n) \equiv [I - B_1 - B_2]^{-1}[A + CG_n] . \tag{6}$$

The rational expectations approach assumes that agents in the economy know the "true structure of the model," i.e., the structures (3) and (4), including exact knowledge of the coefficient matrices A, B_1, B_2, and C. On the other hand, it is recognized [6] that government planners will typically have to estimate A and C in (3) and B_1 and B_2 in (4). The exact functional form of $Q(\cdot)$ in (6) would then have to be estimated, presumably on-line. Traditional econometric techniques are not directly applicable to this task. In Ref. [11] it is suggested that Kalman filtering techniques might be used.

Many critics have faulted the rational expectations assumption that agents other than government have perfect knowledge of the true structural model. However, a second major difficulty with the rational expectations approach which has not received as much attention is that specifications such as (4) for the behavior of "rational" economic agents are ad hoc, since they are not derived from the underlying optimization problems (e.g., profit maximization) which actually face these agents.

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IV. WORKING GROUP REPORTS

Report of the
WORKING GROUP ON ROBUST CONTROL

Discussion Leader: J. Ackermann

I. Presentations

1)	C. L. Nefzger	Jet engine control problems
2)	D. Bowser	Uncertainty of aircraft models
3)	R. Mehra	Model algorithmic control
4)	M. Safonov	Abstract characterization of robustness
5)	D. Young	High gain concepts: Interaction with high frequency modes
6)	J. Ackermann	Parameter space design of robust control
7)	D. Looze	Optimization approach for robust control
8)	R. Marsh	Problems in unstable aircraft

II. Discussions

Impulse response models of the plant -- although not a minimal representation -- offer computational advantages because the output (convolution sum) is bilinear in input and impulse response, i.e. for a given input linear in the model coefficients.

Update reference trajectory during operation to avoid stationary error. Computations with truncated impulse response (e.g. after 20 significant steps), i.e. only for stable plants. Also other techniques assume open loop stability.

Combinations of adaptive and fixed gain robust control:

1. Use a fixed gain robust controller mainly for stabilization, then improve performance by adaptive control.

2. Use a fixed gain robust controller as backup for the case of a failure

in the adaptive system or in a gain scheduling system. Air data measurements, e.g. dynamic pressure, are not very reliable.

3. Under external noise an adaptive system may not adjust fast enough to a fast change in plant parameters (e.g. drop of a load, variation of the geometry, hurt in fight). Switch to a fixed gain robust system, until the identification has followed and adaptation can improve the performance.

4. Adaptive control theory usually does not deal with problems of structural identification (e.g. failure detection) and structural adaptation after a failure has been detected. However problems are related: Fast structural identification may lead to false alarms, in particular under noisy conditions. Slow and reliable structural identification may leave the system in a failed unstable configuration for a while. The control should be designed to provide robustness of stability with respect to the failure, then nothing very bad happens until the failure is detected reliably.

5. Robust fixed gain control may be combined with some redundancy concepts. Various levels are possible:

a) Passive redundancy by paralleled components. The 50% gain reduction margin of LQ designs offers the possibility to use two paralleled sensors or actuators, such that in case of a failure the gain is reduced only by 50%.

b) Removal of failed components. Even if a component failure can be tolerated, as far as stability is concerned, it may be necessary in the long run to remove a failed component, e.g. to close a leaking gas jet valve by a safety valve or to remove a bias term entering into a control system from a sensor failed at a nonzero constant value.

c) Analytic redundancy may help, if an adaptive observer provides an estimate for a missing signal.

d) Hardware redundancy, e.g. majority voting in a multiplexed system can bring the system back to its original performance. However this part of the system ideally should not be vital for stability, see 4.).

6. Examples show that a wide range of parameter variations can be accomodated by a fixed gain robust controller, provided only physically reasonable requirements are made, in particular: Do not try to make a slow system fast or a fast system slow. Not one reference model, but fast and slow reference models, for different operating conditions. Or in robust control: Invariance only of damping or maximum overshoot, not of natural frequency or time of maximum overshoot. Of course it depends on the application which property must be robust: In the design of an oscillator the frequency must be constant, in the design of a crane the frequency of oscillation is unimportant.

If robust control cannot cover the whole range of parameter variation, it may be necessary to use two or more fixed gain controllers and gain scheduling based for example on a dynamic pressure measurement or other crude classification of flight conditions. Here the question of robustness of gain scheduling in view of the use of not very reliable air data arises. Thus it may be a design goal to have a wide overlapping range of flight conditions, for which either set of gains gives satisfactory stability, such that the switching condition is not critical. We have discussed just the first step towards adaptation, with each additional step additional robustness problems will arise.

The Man and the Machine

1. Some requirements on the control system are typical for situations where a man is controlling the outer loop. A man can control also an unstable plant provided the eigenvalues in the right half plane are close to the origin. He has more problems if he has to control fast modes, even if they are slightly damped. In other words, the stability boundary is not necessarily the best emergency boundary for sensor failures, it may be something like the boundary in Fig. 1.

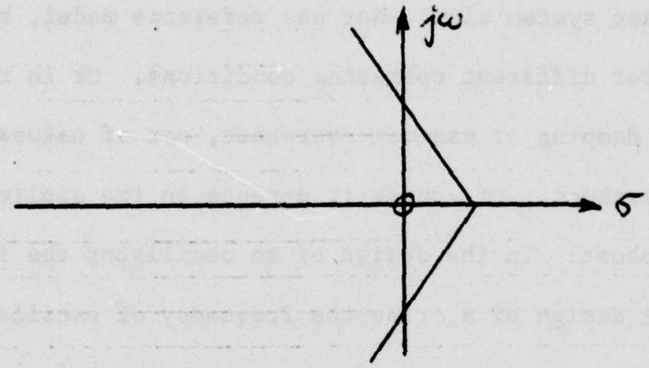

Fig. 1. Emergency boundary for sensor failures in situations with a man in the outer loop.

2. The problems of actuator and sensor failures look similar, if we interpret them as a row or a column of the feedback matrix being switched to zero. However man can use his sensors (eyes, feeling for accelerations) as back up, i.e. for him the aircraft remains observable under sensor failures, however it may become uncontrollable under actuator failures.

3. The pilot does not want to be a passenger. He may want to identify the controlled aircraft by "playing" with the input signals. Control schemes, which give him the same feeling for a wide range of parameter variations, may be dangerous, if the dynamics suddenly become bad beyond an assumed range of parameter variation. The pilot needs a warning before the "cliff." This is another reason why the dynamics should change with changing parameters, i.e.

for different operating points different reference models should be assumed. Previously we had discussed this point only under the aspect of control and control rate limitations.

4. The human is a natural example for the solution of robustness problems for sensors and actuator failures. The fact that we have two eyes and two ears gives us the additional capability of stereo vision and hearing in the unfailed case. In a discussion we use both voice and hands (e.g. writing on a blackboard) to communicate, although we could still communicate if one of these actuators fails. It may be conjectured that the idea of having standby components, which do not contribute to the nominal performance in the unfailed case, are typical only for man-made systems (e.g. redundant components, spare tire).

5. Also the designer is a human. Control theory should provide him with convenient tools, e.g. for the computer-aided design of control systems, instead of demanding that the designer has to put all thinkable tradeoff situations into one scalar performance index or set of inequalities.

A general conclusion by the participants was that this type of workshop with participation of practitioners and theoreticians is very helpful for the mutual understanding and cooperation of both sides. The practitioners emphasized and specified their need for robust control.

Report of the

WORKING GROUP ON MODEL REFERENCE ADAPTIVE CONTROL AND STOCHASTIC SELF-TUNING REGULATORS

Discussion Leader: I. D. Landau

The following subjects were discussed:

1. Deterministic assumptions for the design of MRAC.
2. Stochastic assumptions for the design of STURE.
3. Stability and convergence problems for deterministic and stochastic adaptive control.
4. Transients of the adaptation processes.
5. Explicit and Implicit MRAC.
6. Assumptions upon the leading coefficient of the plant transfer function.
7. Adaptive control of non-minimum phase plants.
8. Use of reduced order models.
9. Suggestions for theoretical research.
10. Suggestions for applications of MRAC and STURE.

We next summarize for each of these subjects the discussions and conclusions of the working group.

I. Deterministic Assumptions for the Design of MRAC

Consider that the plant to be controlled is characterized by the transfer function:

$$T_p(s) = \frac{g\alpha_p(s)}{\beta_p(s)}$$

$$\alpha_p(s) = 1 + \alpha_1 s + \ldots + \alpha_m s^{m_p}$$

$$\beta_p(s) = 1 + \beta_1 s + \ldots + \beta_n s^{n_p} .$$

The following assumptions are considered for the design:

1) Knowledge of a bound n:

$$n > n_p = \deg \beta_p(s) .$$

2) Exact knowledge of n^* in the continuous case:

$$n^* = \deg \beta_p - \deg \alpha_p = n_p - m_p .$$

In the discrete case, the plant delay is the equivalent of n^* and should be known.

3) The design is based on input-output data only.

4) The design does not use differentiators in the continuous case or predictors in the discrete case.

5) The plant is assumed to be minimum phase.

6) The design provides only infinite time convergence results.

7) Some a priori information upon the gain "g" is necessary (for further details, see subject 6).

II. <u>Stochastic Assumptions for the Design of STURE</u>

In addition to the assumptions considered in the deterministic case, the following assumptions are specific for the stochastic case:

1) Existence of disturbances.

2) The colour of disturbances.

3) Only second order properties are necessary for design.

Some particularities have been enhanced namely:

• In the stochastic context, the poles of the observers (or state variable filters) are adapted since their optimal values will depend on the colour of the disturbance. This is not the case for MRAC.

• Large changes in parameters lead to violent transients.

III. Stability and Convergence

In the context of MRAC and STURE, the term stability roughly means that all the quantities are bounded. Convergence means that in the deterministic context, the plant-model error goes to 0 as $t \to \infty$ and in the stochastic context, convergence means that the adjustable parameters converge in probability to the desired values.

Recent research results have established several procedures for proving stability and convergence of MRAC and STURE and these procedures have been already applied to analyze several typical schemes.

These proofs have emphasized the differences between discrete time scheme and continuous time scheme. These differences came essentially from the following:

- In the continuous time case, the output of a minimum phase asymptotically stable linear block can be bounded with the input being unbounded.
- In the discrete time case, the boundedness of the output of a minimum phase asymptotically stable linear block implies the boundedness of the input.

IV. Transients of the Adaptation Processes

A theory for the transients for adaptation processes (bounds) does not yet exist. Very few references on this area are available.

Some work has been done to determine the influence of the various design choices upon the transients:

- Choice of the gain sequence: The basic adaptation algorithms are of the form:

$$\hat{p}(k+1) = \hat{p}(k) + F_k \phi_k e_{k+1} \tag{1}$$

$$\hat{p}(k+1) = \hat{p}(k) + \gamma_k \phi_k e_{k+1} \tag{2}$$

where (1) is also called of L.S. type (F_k is a matrix adaptation gain), and

(2) is called of "stochastic approximation" type (γ_k - scalar adaptation gain). For (1), the gain can be updated by the general formula:

$$F_{k+1}^{-1} = \lambda_1(k)F_k^{-1} + \lambda_2(k)\phi_k\phi_k^T$$

$$0 < \lambda_1(k) \leq 1 \quad ; \quad 0 \leq \lambda_2(k) < 2 \quad ; \quad F_0 > 0$$

and the transients will depend upon the choice of the $\lambda_1(k)$ and $\lambda_2(k)$. For (2), the basic formula is $\gamma_k = \frac{1}{k}$, however modification of this rule at the beginning of the adaptation is highly beneficial (instead of γ_k such that $k\gamma_k = 1$, for all k, one uses another sequence γ_k such that $k\gamma_k$ starts at 1, grows to 2 and then returns to 1).

- Choice of the strictly positive real transfer functions.

The designs require that a certain transfer function:

$$H(z^{-1}) = \frac{D(z^{-1})}{A(z^{-1})}$$

be strictly positive real where $A(z^{-1})$ is known and $D(z^{-1})$ should be calculated in order to satisfy the S.P.R. condition. The transients will depend on the particular choice of $D(z^{-1})$.

- Proportional + integral adaptation: Adding proportional adaptation when possible, the transients can be improved.

The study of the optimization of the gain profile and of the S.P.R. choice is very important for assuring high performances for the adaptive control schemes.

V. Explicit and Implicit MRAC

Several terms are used to designate these schemes:

- Explicit MRAC = Direct MRAC.
- Implicit MRAC = Indirect MRAC.

Implicit MRAC belongs also to the class of adaptive control systems which use as an intermediate step an adaptive predictor estimator. This kind of scheme is potentially richer than explicit MRAC. However stability and convergence results are available only for those schemes which are equivalent to explicit MRAC.

VI. Assumptions on the Leading Coefficient of the Plant Transfer Function

The plant transfer function considered is:

$$T_p = \frac{g\alpha_p(s)}{\beta_p(s)} .$$

The designs presented in the literature can handle several cases:

a) g is known,

b) the sign of g is known and a bound (inferior and/or superior),

c) g is unknown but of constant sign.

The problems which should be considered in more detail are:

- zero crossing during adaptation transients, and

- change in sign of g.

The most difficult problem is the change of sign of g and "cautious" or "dual" techniques should be used in order to avoid large transients.

VII. Adaptive Control of Non-Minimum Phase Plants

Not any technique fully satisfactorily for handling this type of problem is available. The available techniques are basically three.

1. Explicit identification of the zeros and polynomial division. Leads to numerical problems.

2. One step ahead criterion optimization (Clarke, Gathrop, Johnson). Can not stabilize any plant.

3. Implicit MRAC using an adaptive observer in "controllable canonical form" or Explicit MRAC with an evolutive reference model (Silveira). The plant should be stable or stabilizable by an output constant feedback for all the possible values of plant parameters.

VIII. <u>Reduced Order Models</u>

The reduced order models enter in the adaptive control problem because the plant transfer function considered in the design is either an approximation done because a part of the dynamics can be neglected in open loop or one would like to have a certain controller complexity.

The adaptive control scheme features two time scale phenomena allowing a separation and this separation can be parametrized in terms of a particular μ. The approximate design corresponds to $\mu = 0$ and the use of singular perturbation techniques will allow to measure the validity of this approximation.

The need for implementation of the transient adaptation signals can also be probably examined within the same context.

IX. <u>Suggestions for Theoretical Research</u>

The subjects which are listed below have been considered not only as being very useful for the development of MRAC-STURE techniques but the basis are available and therefore the research can be started immediately.

1. Theoretical analysis of adaptation transients.
2. Analysis under disturbances.
3. Adaptive control of non-minimum phase plant.
4. Stability and convergence analysis of certain known schemes.
5. Parametrization studies for control of M.I.M.O. system in view of the use of adaptive control.
6. Adaptive control of M.I.M.O. systems.

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7. Design of adaptive control schemes for restricted domain of parameter variations.

8. Design of adaptive control schemes using adjustable reference models.

9. Effects of saturation on the control and its derivative.

10. Adaptive control of time varying plants.

11. Adaptive control of certain classes of non-linear plants.

12. Analysis of adaptive control schemes using reduced order models.

13. Use of singular perturbation methods for MRAC analysis.

14. Robustness of adaptive control schemes with respect to design assumptions.

15. Loop gain and frequency response interpretation of the adaptive loop.

16. How to take in account the available information for the design of adaptive control schemes.

17. Development of alternative adaptation schemes.

18. The analysis of "subsystem" type effects upon adaptive control scheme (ex: adaptive voltage control in power systems).

The order of the various research subjects does not represent an order of preference. However, with respect to the potential demand of users for adaptive control, the first six subjects should be considered with a certain priority.

X. Suggestions for Applications

The discussions were oriented essentially with respect to applications in aeronautics and astronautics.

It appeared that the application of MRAC-STURE techniques to the C|3| variable stability aircraft is feasible and will allow an extensive evaluation

under various conditions. It was pointed out that linear model following control has been already implemented on this aircraft.

Other possible applications include: space shuttle, missiles, jet engines, pointer systems, submarines. In addition, studies concerning the potential use of MRAC-STURE for reconfiguration of control after fault and for high angle of attack aircraft will be useful.

Several successful applications of MRAC-STURE techniques for the control of various industrial (or pilot) processes have been done in the last 5 years and mentioned during the discussions. However these techniques have not yet become standard techniques and more effort should be done in this direction.

Report of the

WORKING GROUP ON STOCHASTIC ADAPTIVE CONTROL

Discussion Leader: Y. Bar-Shalom

1. Role of Stochastic Adaptive Control

Sources of uncertainty: parameters ("slowly" varying states), process and measurement disturbance.

If (1) a loss function is used as performance measure and (2) the uncertainties are modeled probabilistically then only Stochastic Control methodology can evaluate the performance degradation due to each (and all) uncertainties--which parameters are critical to control performance (A vs. B).

Meaning of expected value of loss function

• minimum variance (1 step horizon).

• general - N step horizon, weighted for state components and control.

The stochastic control approach yields a control law that depends on

(1) current information (adaptivity),

(2) quality of the current information, and

(3) anticipated quality of future information.

Other approaches do not incorporate (2), (3).

Stochastic control methodology can be used to characterize conditions when simpler algorithms are adequate:

• when some uncertainties can be neglected.

• when the results of an identification procedure are adequate for the control purpose (identification accuracy requirements should be determined by the control problem where the model is used).

In the overall system optimization

• the estimation has to serve the control purpose.

• the control has in general a dual purpose (1) control proper and (2) estimation enhancement.

Therefore simultaneous optimization is needed.

In some missile guidance problems sophisticated (but not necessarily expensive) control can enhance estimation quality or reduce sensor requirements.

Such guidance can yield range information from angle only sensors. This (1) reduces missile distance and (2) allows adaptive estimation (noise covariance is r-dependent).

Adaptive stochastic control can be used to guarantee (with probability 1) stability of an unknown system with disturbances under certain conditions. This is an alternate approach to minimization of expected loss.

2. Existing Stochastic Control Algorithms

A. Continuous valued parameters

HCE - uses parameter estimates in place of the true values. If uncertainties are "small" it can be adequate. Stochastic analysis is required to reach this conclusion without extensive simulations.

STURE - one step look-ahead minimum variance. Simplest, and extensively studied implemented for process control.

Modified STURE - to include also one step ahead estimation enhancement (via dual effect).

SAFER (sensitivity adaptive feedback with estimation redistribution) - exploits sensitivity to improve the estimation for the control purpose.

WSADC - approximation of the dynamic programming that yields explicit expressions of the stochastic effects (divided into caution and probing terms).

Min-max increment algorithm - minimizes the largest performance degradation within the range of unknown parameters.

B. Discrete valued parameters

HCE (heuristic certainty equivalence) - uses parameter estimates in place of the true values. If uncertainties are "small" it can be adequate. Stochastic analysis is required to reach this conclusion without extensive simulations.

MMAC (partition) algorithm - model-optimum controls weighted by their a posteriori probabilities.

MAD - model adaptive dual - has the identification enhancement feature.

3. Topics for Future Research

- Nonlinear stochastic control problems - only specific classes of problems can be solved, e.g. parametric imbedding of nonlinear problems.
- Convergence (and rate of convergence) of stochastic adaptive control algorithms and stability of the overall system.
- Use of robust structures to initialize the adaptation process; tradeoff between robust structures and accuracy needs of adaptation.
- Development of meaningful methodology for comparison of stochastic algorithms.
- Interplay of model choice and control law.
- Modeling of sources of uncertainty (Markov, others?).
- Development of tools for non-Monte Carlo quantitative assessment of the performance improvement obtainable by a stochastic adaptive control approach.
- Development of actively adaptive (estimation enhancing) control algorithms for new classes of problem - missile guidance, ECM.

- Probing - Caution interrelationship.
- Simplified actively adaptive algorithms.
- Relationship among the various existing schemes (Bayesian vs. min-max).

V. SUMMARY AND CONCLUSIONS

The adaptive control field exists because of a need to achieve high system performance in spite of severe variations in the characteristics of the process to be controlled and in spite of a wide range of operating and environmental conditions. Two complementary approaches to the representation of uncertainty, deterministic and stochastic, are useful. Both characterizations are employed in the investigation of fixed feedback structures and the investigation of adaptive controllers which automatically adjust to changes in the process characteristics. Fixed controllers which insure satisfactory operation in spite of wide variations in process characteristics are called robust controllers. The most common adaptive controllers are the model-reference adaptive controller and the self-tuning regulator. The most demanding adaptive control problem is stochastic with a general performance criterion containing several stages.

Whenever robust controls can perform satisfactorily in solving adaptive control problems, they offer an attractive alternative to active adaptive controls because of their relative simplicity in realization and hence greater reliability. Even when adaptive controls are to be used, the robust controller is a useful backup to provide adequate control for emergency conditions. Furthermore, adaptive controllers which are also robust would be desirable. Concepts from robust control theory would enrich the field of active adaptive control. Several important conclusions regarding robust controls are listed in Section IV-A.

Model reference adaptive controls have been investigated for almost twenty years now and many results are ready for applications. Many of the recent results are listed in the references of Section III-B. One important

recent result concerns stability and convergence. Because of the focus on this well-defined problem area, several specific suggested areas for future research resulted from the discussion group. These are listed in Section IV-B. Several of these could be expected to be solved in the near future.

In the area of general stochastic adaptive control, the problem is very difficult and most results to date are quite involved. An important exception is the self-tuning regulator which is developed largely for single-input single-output linear systems with minimization of a one-step ahead mean-square error. This has been developed over the last twelve years and it has been successfully applied to industrial process control problems. Other special classes of stochastic adaptive control problems are under investigation and the researchers in the field are seeking practical control algorithms. Several important application problems were discussed in the working group and several suggested directions for research are listed in Section III-C.

From the discussions of all three working groups, it was clear that concepts of robust control could significantly enrich the study of adaptive control. Adaptive controllers should have some robustness properties so that the adaptation would not have to be too critical. In the context of stochastic control, the estimation aspect of the control can be simpler if the control is robust. Moreover, concepts of sensitivity which are deeply rooted in feedback theory can be exploited in stochastic controllers with dual effects.

The size of the workshop and the particular mix among industrial representatives, academic researchers, and government laboratory scientists and engineers appeared to be just right. The informal arrangement and organization encouraged much discussion, interchange of ideas, and stimulation. There was much enthusiasm for further research in adaptive control.

APPENDIX A

AFOSR WORKSHOP ON ADAPTIVE CONTROL

University Inn, Champaign, Illinois
May 8-10, 1979

Agenda

May 8, 1979 — Plaza Room, 8:30 a.m.-5:00 p.m.

8:30 a.m.	Registration
8:45 a.m.	Opening Remarks, Major C. L. Nefzger, AFOSR
9:00 a.m.	Overview of Model Reference Adaptive Control and Self-Tuning Regulators, I. D. Landau, Laboratoire d'Automatique de Grenoble, ENSIEG, France
10:00 a.m.	Coffee
10:30 a.m.	Overview of Stochastic Adaptive Control, Y. Bar-Shalom, University of Connecticut
11-30 a.m.-12:30 p.m.	Overview of Robust Control, J. Ackermann, Institute for Dynamics of Flight Systems, DFVLR, Germany
2:00 p.m.	General Discussion
3:00 p.m.	Coffee
3:30 p.m.	General Discussion
4:30-5:00 p.m.	Organization of Working Groups Model Reference Adaptive Control, Plaza Room A Stochastic Adaptive Control, Plaza Room B Robust Control, Plaza Room C

May 9, 1979

Working Group on Model Reference Adaptive Control and Self-Tuning Regulators, I. D. Landau, Discussion Leader

Working Group on Stochastic Adaptive Control, Y. Bar-Shalom, Discussion Leader

Working Group on Robust Control, J. Ackermann, Discussion Leader

These groups will meet in Plaza Rooms A, B, and C. The meeting rooms will be open 8:30 a.m. - 5:00 p.m. Upon request they may be open in the evenings for extended discussions.

May 10, 1979 — Plaza Room, 8:00 a.m.-5:00 p.m.

8:00 a.m.	Report of Working Group A, I. D. Landau
8:30 a.m.	Report of Working Group B, Y. Bar-Shalom
9:00 a.m.	Report of Working Group C, J. Ackermann
9:30-10:30 a.m.	General Discussion and Concluding Statements
10:45 a.m.	Workshop ends.

APPENDIX B

AFOSR WORKSHOP ON ADAPTIVE CONTROL

Champaign, Illinois, May 8-10, 1979

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